Let X be a continuous random variable with pdf $f(x)$.

EXPECTED VALUE OF X: $\mu = E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$ — values, densities — discrete: $\sum_x x \cdot p(x)$

... OF $h(X)$: $E(h(X)) = \int_{-\infty}^{\infty} h(x) f(x)\,dx$ — $\sum_x h(x) p(x)$

SUMS $\Longrightarrow$ INTEGRALS

VARIANCE OF X: $\mathrm{Var}(X) = E\left[(X-\mu)^2\right] = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$

$$\mathrm{Var}(X) = E(X^2) - E(X)^2$$

MOMENT GENERATING FUNCTION: $M_X(t) = E(e^{tX}) = \int_{-\infty}^{\infty} e^{tx} f(x)\,dx$ — values, densities

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PROBLEM 2:

$X \sim \mathrm{Unif}[A,B]$

$$M_X(t) = E(e^{tX}) = \int_A^B e^{tx} \frac{1}{B-A}\,dx = \left[\frac{1}{t} e^{tx} \cdot \frac{1}{B-A}\right]_{x=A}^{x=B} = \frac{e^{Bt} - e^{At}}{t(B-A)}$$

$A \le x \le B$ — values — density — if $t \neq 0$

If $t=0$: $E(e^{0 \cdot x}) = E(1) = 1$

mgf of $\mathrm{Unif}[A,B]$

$$\boxed{M_X(t) = \begin{cases} \dfrac{e^{Bt} - e^{At}}{t(B-A)} & \text{if } t \neq 0 \\ 1 & \text{if } t = 0 \end{cases}}$$

From #1: $U \sim \mathrm{Unif}[0,5]$

if $t \neq 0$ $\quad M_U(t) = \dfrac{e^{5t} - e^{0t}}{t(5-0)} = \dfrac{e^{5t} - 1}{5t}$

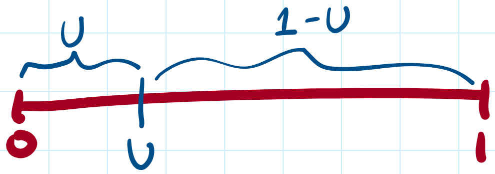
$V = 3U + 2$

$M_V(t) = e^{2t} M_U(3t)$

So: $M_X(t) = e^{2t} M_U(3t) = e^{2t} \dfrac{e^{5(3t)} - 1}{5(3t)}$

$$M_V(t) = \frac{e^{17t} - e^{2t}}{15t}$$ mgf of Unif[2,17] so $V \sim \text{Unif}[2,17]$

PROBLEM #3:



$U \sim \text{Unif}[0,1]$

$E(U) = \frac{1}{2}$