Law of Total Expectation:

$$E(\underbrace{E(X|Y)}_{\text{inside: conditional expected value}}) = E(X)$$

outside: expected value as a function of Y

Law of Total Variance:

$$\mathrm{Var}(X) = \mathrm{Var}(E(X|Y)) + E(\mathrm{Var}(X|Y))$$

From last class:

4. The number of eggs N found in a nest of a certain species of turtle has a Poisson distribution with mean λ. Each egg has a probability p of being viable, and this event is independent from egg to egg. Find the mean and variance of the $\underbrace{\text{number of viable eggs per nest}}_{X}$.

$N \sim \text{Poisson}(\lambda)$ $\qquad$ $X \sim \text{Binomial}(N, p)$

$$E(X) = E(\underbrace{E(X|N)}) = E(Np) = p \cdot E(N) = \boxed{p \cdot \lambda} = E(X)$$

$E(X|N) = Np$

$$\mathrm{Var}(X) = \mathrm{Var}(\underbrace{E(X|N)}_{Np}) + E(\underbrace{\mathrm{Var}(X|N)}_{Np(1-p)}) = \mathrm{Var}(Np) + E(Np(1-p))$$

$$= p^2 \underbrace{\mathrm{Var}(N)}_{\lambda} + p(1-p)\underbrace{E(N)}_{\lambda} = p^2\lambda + p(1-p)\lambda = \cancel{p^2\lambda} - \cancel{p^2\lambda} + p\lambda = \boxed{p\lambda}$$

$\mathrm{Var}(X)$

Central Limit Theorem

Let $X_1, X_2, \ldots, X_n$ be iid rvs with mean μ and standard deviation σ.

Let $T_n = X_1 + \cdots + X_n$ and $\overline{X}_n = \frac{T_n}{n}$.

Then, as $n \to \infty$:

- The distribution of T_n approaches $N(n\mu, \sigma\sqrt{n})$.
- The distribution of $\overline{X}_n$ approaches $N(\mu, \frac{\sigma}{\sqrt{n}})$.