

Warm-Up: A hat contains 3 cards, identical in form, except that both sides of one card are red, both sides of another card are blue, and the third card contains one blue and one red side. One card is randomly selected from the hat and placed on a table. If the visible side of the chosen card is red, what is the probability that the other side of that card is also red?

Events:
 A: red-red card is chosen
 B: blue-blue card is chosen
 C: red-blue card is chosen
 R: visible side of card is red

Want: $P(A | R) = \frac{P(A \cap R)}{P(R)} = \frac{P(R|A)P(A)}{P(R)}$ ← Bayes' Formula

↑
def. of conditional prob.

$$= \frac{1 \cdot \frac{1}{3}}{\frac{1}{2}} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

Law of Total Probability

$$\begin{aligned} P(R) &= P(R|A)P(A) + P(R|B)P(B) + P(R|C)P(C) \\ &= 1 \cdot \frac{1}{3} + 0 \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{1}{3} \\ &= \frac{2}{6} + \frac{1}{6} = \frac{1}{2} \end{aligned}$$