

Warm-Up: A red die and a blue die are rolled. Let A be the event that the red die rolls 2, let B be the event that the sum of the rolls is 5, and let C be the event that the sum of the rolls is 7. Are A and B independent events? How about A and C ?

A and B : not independent

$$\bullet P(A) = \frac{1}{6}, \quad P(B) = \frac{4}{36} \quad P(A \cap B) = \frac{1}{36}$$

$$P(A)P(B) = \frac{1}{6} \cdot \frac{4}{36} \neq \frac{1}{36} = P(A \cap B)$$

• If you know the sum is 5, then neither die can roll a 5 or a 6, so this affects your assessment of the probabilities of the rolls of the red die.

$$\bullet P(A|B) = \frac{1}{4} \quad \text{so} \quad P(A|B) = \frac{1}{4} \neq \frac{1}{6} = P(A)$$

A and C : independent!

$$P(C) = \frac{6}{36} = \frac{1}{6}$$

$$P(A)P(C) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} = P(A \cap C)$$

$$P(A|C) = \frac{1}{6} = P(A)$$