1. A painter has six cans of paint, each containing a different color. Two of the cans contain paint with a satin finish, and four contain glossy paint.

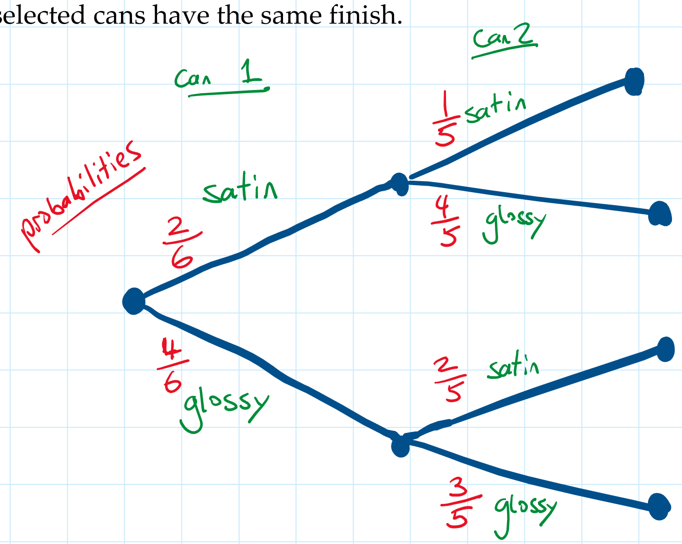
(a) If the painter selects one can of satin paint and one can of glossy paint, how many different color combinations are possible? How does this relate to the Fundamental Counting Principle?

2 choices for the satin paint
4 choices for the glossy paint

$2 \cdot 4 = 8$ possible choices for satin and glossy selections

This IS the fundamental counting principle.

(b) Suppose the painter forgets that the cans contain paint with different finishes, and simply selects two cans at random. Use a tree diagram to help you find the probability that the two selected cans have the same finish.



2 satin paints: $\frac{2}{6}\cdot\frac{1}{5} = \frac{2}{30} = P(\text{both satin})$

satin, glossy $\frac{2}{6}\cdot\frac{4}{5} = \frac{8}{30}$

glossy, satin $\frac{4}{6}\cdot\frac{2}{5} = \frac{8}{30}$

Sum: $\frac{8}{15}$

2 glossy paints $\frac{4}{6}\cdot\frac{3}{5} = \frac{12}{30} = P(\text{both glossy})$

$P(\text{both same finish}) = P(\text{both satin}) + P(\text{both glossy}) = \frac{2}{30} + \frac{12}{30} = \frac{14}{30} = \frac{7}{15}$

NOTE: AND vs OR in counting and probability

AND: corresponds to multiplication; intersection of sets

OR: corresponds to addition; union of sets

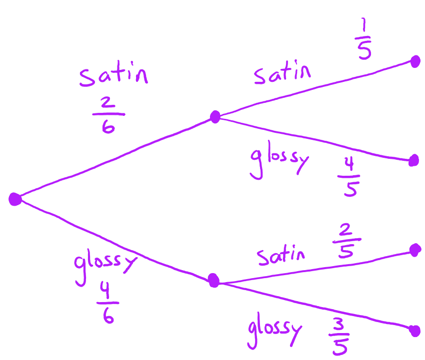
1. A painter has six cans of paint, each containing a different color. Two of the cans contain paint with a satin finish, and four contain glossy paint.

   (a) If the painter selects one can of satin paint and one can of glossy paint, how many different color combinations are possible? How does this relate to the Fundamental Counting Principle?

   (2 possible satin paints) $\cdot$ (4 possible glossy paints) $=$ 8 possible pairs

   This is the fundamental counting principle: selecting one item in 4 ways, followed by another item in 2 ways, gives $4 \cdot 2 = 8$ possible selections.

   (b) Suppose the painter forgets that the cans contain paint with different finishes, and simply selects two cans at random. Use a tree diagram to help you find the probability that the two selected cans have the *same* finish.



   $\frac{2}{6} \cdot \frac{1}{5} = \frac{2}{30} = P(\text{both satin})$

   $\frac{2}{6} \cdot \frac{4}{5} = \frac{8}{30}$

   $\frac{4}{6} \cdot \frac{2}{5} = \frac{8}{30}$

   $\frac{4}{6} \cdot \frac{3}{5} = \frac{12}{30} = P(\text{both glossy})$

   $P(\text{same finish}) = P(\text{both satin}) + P(\text{both glossy}) = \frac{2}{30} + \frac{12}{30} = \frac{14}{30} = \frac{7}{15}$

2. Minnesota issues license plates that consist of three numbers followed by three letters; for example: 012-ABC. How many different license plates are possible?

   Choose the numbers in 10^3 ways, and the letters in 26^3 ways.

   By the FCP, the total number of choices is $10^3 \cdot 26^3 = 17{,}576{,}000$.

3. How many different 4-letter codes can be made from the letters in the word *PADLOCKS*, if no letter can be chosen more than once? How about 6-letter codes from the letters in *DOGWATCHES*?

   "PADLOCKS" has 8 distinct letters: ${}_8P_4 = \frac{8!}{(8-4)!} = 1680$

"DOGWATCHES" has 10 distinct letters: $_{10}P_6 = \frac{10!}{(10-6)!} = 151{,}200$

4. In a certain lottery, players select six numbers from 1 to n. For each drawing, balls numbered 1 to n are placed in a hopper, and six balls are drawn at random and without replacement. To win, a player's numbers must match those on the balls, in any order.

(a) If $n = 15$, how many combinations of winning numbers are possible?

$$\binom{15}{6} = \frac{15!}{9!\,6!} = 5005$$

(b) If $n = 24$, how many combinations of winning numbers are possible?

$$\binom{24}{6} = \frac{24!}{18!\ 6!} = 134{,}596$$

(c) If $n = 24$, what is the probability that the six balls that are drawn contain only numbers less than 16?

$$\frac{5005}{134596} = 0.037$$

(d) If $n = 24$, what is the probability that the ball numbered 8 is among the balls drawn?

If 8 is drawn, then there are $\binom{23}{5}$ ways to choose the other 5 balls.

Thus, the probability that 8 is drawn is:

$$\frac{\binom{23}{5}}{\binom{24}{6}} = \frac{\frac{23!}{18!\,5!}}{\frac{24!}{18!\,6!}} = \frac{23!\,6!}{24!\ 5!} = \frac{6}{24} = \frac{1}{4}$$

5. An absent-minded secretary prepared five letters and envelopes addressed to five different people. The secretary placed the letters randomly in the envelopes. A match occurs if a letter and its envelope are addressed to the same person. What is the probability of the following events?

(a) All five letters and envelopes match.

There are $5! = 120$ permutations of letters in envelopes, only one of which results in all five matches.

Thus, the probability of five matches is $\frac{1}{120}$.

(b) Exactly four of the five letters and envelopes match.

This is not possible — if four envelopes and letters match, then the fifth must match also. The probability is zero.

(c) **BUNUS:** None of the letters and envelopes match.

This is a hard problem. It won't be on the exam!

Let A_i be the event that the i^{th} letter and envelope match.

So: $P(A_i) = \frac{4!}{5!} = \frac{1}{5}$ ← since 4 letters may be assigned arbitrarily

$P(A_i \cap A_j) = \frac{3!}{5!} = \frac{1}{20}$ ← since 3 letters may be assigned arbitrarily

etc.

The probability of some match is:

$$P(A_1 \cup A_2 \cup \cdots \cup A_5) = \sum_i P(A_i) - \sum_{i,j} P(A_i \cap A_j) + \sum_{i,j,k} P(A_i \cap A_j \cap A_k) - \sum_{i,j,k,\ell} P(A_i \cap A_j \cap A_k \cap A_\ell) + P(A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5)$$

inclusion-exclusion principle

$$= 5 \cdot \frac{1}{5} - \binom{5}{2}\frac{1}{20} + \binom{5}{3}\frac{1}{60} - \binom{5}{4}\frac{1}{120} + \frac{1}{120}$$

$$= \frac{19}{30}$$

Thus, the probability of no match is $1 - \frac{19}{30} = \boxed{\frac{11}{30}}$.