BINOMIAL COEFFICIENTS:

We've said $\binom{n}{k}$ is "n choose k" — the number of ways of selecting k items from n possibilities (without replacement, order unimportant).

COMBINATIONS

Consider: $(a+b)^3 = (a+b)(a+b)(a+b) = \binom{3}{3}a^3 + \binom{3}{2}a^2b + \binom{3}{1}ab^2 + \binom{3}{0}b^3$

$= 1a^3 + 3a^2b + 3ab^2 + 1b^3$

Similarly, $(a+b)^n = \binom{n}{n}a^n + \binom{n}{n-1}a^{n-1}b + \cdots + \binom{n}{0}b^n$

They also occur in Pascal's triangle:

$$\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}
\qquad
\begin{array}{ccccccc}
 & & & \binom{0}{0} & & & \\
 & & \binom{1}{0} & & \binom{1}{1} & & \\
 & \binom{2}{0} & & \binom{2}{1} & & \binom{2}{2} & \\
\binom{3}{0} & & \binom{3}{1} & & \binom{3}{2} & & \binom{3}{3} \\
 & & & \vdots & & &
\end{array}$$

Why? $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

Selecting k items out of n	ORDER IMPORTANT	ORDER NOT IMPORTANT
WITH REPLACEMENT	n^k	$\binom{k+n-1}{k} = \frac{(k+n-1)!}{(n-1)!\ k!}$
WITHOUT REPLACEMENT	$\frac{n!}{(n-k)!}$ permutation	$\binom{n}{k} = \frac{n!}{(n-k)!\,k!}$ combination