MATH 262

Section 1.5 Day 6

1. A sequence of n independent trials are to be performed. Each trial results in a success with probability p and a failure with probability $1-p$. What is the probability that...

   (a) ...all trials result in successes?

   (b) ...at least one trial results in a success?

   (c) ...exactly k trials result in successes?

2. Consider an urn containing four balls, labeled 110, 101, 011, and 000. One ball is drawn at random. For $k \in \{1,2,3\}$, let A_k be the event that the k^{th} digit is a 1 on the ball that is drawn.

   (a) Are the events A_1, A_2, and A_3 pairwise independent? Why or why not?

   (b) Are the events A_1, A_2, and A_3 mutually independent? Why or why not?

3. If A and B are independent events each with positive probability, show that they cannot be mutually exclusive.

4. Create an example of three events A, B, C such that $P(A \cap B \cap C) = P(A)P(B)P(C)$ but the events are not mutually independent. (One way to do this is to draw a Venn diagram, specifying probabilities of A, B, C and their intersections.)

★ **BONUS:** Suppose that A and B are independent events, and event C is such that $P(C) > 0$. Is it true that $P(A \cap B \mid C) = P(A \mid C)P(B \mid C)$?