

MATH 262

Sections 2.1 and 2.2

Day 8

1. Let random variable X be the output of `runif(1)` in **R** (or, if you prefer, the output of `RandomReal[]` in Mathematica). Is X a continuous or discrete random variable?

(a) Have one person in your group defend the assertion that X is a continuous random variable.

(b) Have another person in your group defend the assertion that X is a discrete random variable.

2. The cdf for a random variable X is as follows:

$$F(x) = \begin{cases} 0 & x < 0 \\ 0.2 & 0 \leq x < 1 \\ 0.5 & 1 \leq x < 2 \\ 0.8 & 2 \leq x < 4 \\ 1 & 4 \leq x \end{cases}$$

(a) What is $P(X = 2)$?

(b) What is $P(X = 3)$?

(c) What is $P(2.5 \leq X)$?

(d) Sketch the pmf of X .

3. Each of the following functions might be the pmf for some random variable X . *Discuss with your group:* How can you determine whether a given function is a pmf? Which of these functions is a pmf?

(a) $p(x) = 2 - 3x$ for $x = 0, 1$

(b) $p(x) = \frac{x^2}{50}$ for $x = 1, 2, \dots, 5$

(c) $p(x) = \log_{10}\left(\frac{x+1}{x}\right)$ for $x = 1, 2, \dots, 9$

4. Which of the following properties must hold for any cdf $F(x)$? Discuss each property with your group. Either say why it must hold or give a counterexample to show that it might not hold.

(a) $\lim_{b \rightarrow -\infty} F(b) = 0$

(b) $\lim_{b \rightarrow \infty} F(b) = 1$

(c) $F(x)$ is continuous

(d) $F(x)$ is nondecreasing; that is, if $a < b$, then $F(a) \leq F(b)$

(e) $F(b) = 0.5$ for some value b

★ **BONUS:** Three balls are randomly selected (without replacement) from an urn containing 20 balls numbered 1 through 20. Let random variable X be the largest of the three selected numbers. What is $P(X = 17)$? What is $P(X \geq 17)$?