1. Let random variable X be the output of `runif(1)` in **R** (or, if you prefer, the output of `RandomReal[]` in *Mathematica*). Is X a continuous or discrete random variable?

(a) Have one person in your group defend the assertion that X is a continuous random variable.

X is equally likely to take any value between 0 and 1, so X is a continuous random variable.

(b) Have another person in your group defend the assertion that X is a discrete random variable.

X is specified by a finite number of decimal digits, so there are only finitely many possible values for X, which makes X a discrete random variable.

How do we reconcile these two assertions?

X is a discrete approximation of a continuous random variable.

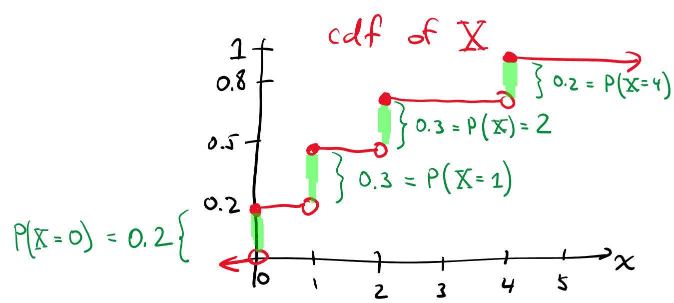
2. The cdf for a random variable X is as follows:

$$X = \begin{cases} 0 & x < 0 \\ 0.2 & 0 \le x < 1 \\ 0.5 & 1 \le x < 2 \\ 0.8 & 2 \le x < 4 \\ 1 & 4 \le x \end{cases}$$

(a) What is $P(X = 2)$?

largest possible value of X that is strictly less than 2

$P(X=2) = F(2) - F(2-) = 0.8 - 0.5 = 0.3$



(b) What is $P(X = 3)$?

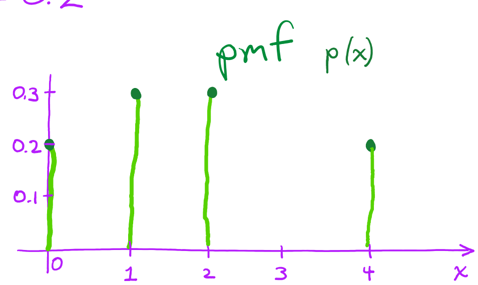
$P(X=3) = F(3) - F(3-) = 0.8 - 0.8 = 0$

(c) What is $P(2.5 \le X)$?

$P(2.5 \le X) = 1 - F(2.5-) = 1 - 0.8 = 0.2$

(d) Sketch the pmf of X.

The nonzero values of $p(x)$ are $p(0) = 0.2$, $p(1) = 0.3$, $p(2) = 0.3$, and $p(4) = 0.2$.



3. Each of the following functions might be the pmf for some random variable X. How can you determine whether a given function is a pmf? Which of these functions is a pmf?

(a) $p(x) = 2 - 3x$ for $x = 0,1$

This is not a pmf because $p(1) = -1$,
but probabilities must be nonnegative.

(b) $p(x) = \frac{x^2}{50}$ for $x = 1, 2, \dots, 5$

This is not a pmf because $\sum_{x=1}^{5} \frac{x^2}{50} = \frac{1+4+9+16+25}{50} = \frac{55}{50} \neq 1$

(c) $p(x) = \log_{10}\left(\frac{x+1}{x}\right)$ for $x = 1, 2, \dots, 9$

Since $p(x) \geq 0$ for $x = 1, 2, \dots, 9$ and
and $\sum_{x=1}^{9} \log_{10}\left(\frac{x+1}{x}\right) = \log_{10}\left(\frac{2}{1}\cdot\frac{3}{2}\cdot\frac{4}{3}\cdot \dots \cdot\frac{10}{9}\right) = \log_{10}(10) = 1$,
this is a pmf.

(This is the pmf of the distribution known as Benford's Law.)

4. Which of the following properties must hold for any cdf $F(x)$? For each property, either say why it must hold or give a counterexample to show that it might not hold.

(a) $\lim_{b\to-\infty} F(b) = 0$

Yes, since $P(X \leq b) \to 0$ as $b \to -\infty$.

(b) $\lim_{b\to\infty} F(b) = 1$

Yes, since $P(X \leq b) \to 1$ as $b \to \infty$.

(c) $F(x)$ is continuous

No — consider $F(x)$ in #2 above.

(d) $F(x)$ is nondecreasing; that is, if $a < b$, then $F(a) \leq F(b)$

Yes, if $a < b$, then

$$F(a) = P(X \leq a) \leq P(X \leq a) + \underbrace{P(a < X \leq b)}_{\text{this is nonnegative}} = P(X \leq b) = F(b)$$

(e) $F(b) = 0.5$ for some value b

No — consider the cdf in #2 above.

BONUS: Three balls are randomly selected (without replacement) from an urn containing 20 balls numbered 1 through 20. Let random variable X be the largest of the three selected numbers. What is $P(X = 17)$? What is $P(X \geq 17)$?

Assume that each of the $\binom{20}{3}$ selections are equally likely.

The event $(X \leq x)$ occurs when the three selected balls have numbers less than or equal to x. There are $\binom{x}{3}$ ways to select three such balls. Thus,

$$F(x) = P(X \leq x) = \frac{\binom{3}{x}}{\binom{20}{3}}.$$

We then compute the desired probabilities:

$$P(X = 17) = F(17) - F(16) = \frac{\binom{17}{3}}{\binom{20}{3}} - \frac{\binom{16}{3}}{\binom{20}{3}} = \frac{2}{19} \approx 0.105$$

$$P(X \geq 17) = 1 - F(17) = 1 - \frac{\binom{17}{3}}{\binom{20}{3}} = \frac{29}{57} \approx 0.509$$