Recent homework assignments have asked you to read several articles about the concept of a growth mindset.

- What have you found most insightful or impactful from these articles?
- How could you cultivate a growth mindset, and how could this help you succeed in this course?

Warm-up: Define the following random variables.

$X = 0$ with probability 1.

$E(X) = 0(1) = 0$

$Y = \pm 1$ with probability $\frac{1}{2}$ each.

$E(Y) = 1\left(\frac{1}{2}\right) + (-1)\left(\frac{1}{2}\right) = 0$

$Z = \pm 100$ with probability $\frac{1}{2}$ each.

$E(Z) = 100\left(\frac{1}{2}\right) + (-100)\left(\frac{1}{2}\right) = 0$

1. Which of the following is true?

   (a) $E(X) = E(Y) = E(Z)$ ⟵ True   since $E(X) = E(Y) = E(Z) = 0$
   (b) $E(X) < E(Y) < E(Z)$
   (c) $E(X) > E(Y) > E(Z)$

$Var(Z) = E(Z^2) - \left(E(Z)\right)^2 = 10000 - 0$

$E(Z^2) = (100^2)\left(\frac{1}{2}\right) + \left((-100)^2\right)\left(\frac{1}{2}\right) = 10000$

2. Which of the following is true?

   (a) $Var(X) = Var(Y) = Var(Z)$ $V(X) = 0$
   (b) $Var(X) < Var(Y) < Var(Z)$
   (c) $Var(X) > Var(Y) > Var(Z)$ $V(Y) = 1$

Chebyshev's Inequality: Let X be a discrete random variable with mean μ and standard deviation σ. For any $k \geq 1$,

$$P\left(|X - \mu| \geq k\sigma\right) \leq \frac{1}{k^2}.$$

In words, the probability that X is at least k standard deviations away from its mean is at most $\frac{1}{k^2}$.

Prob. that X is in the orange region is $\leq \frac{1}{k^2}$

$k\sigma$ μ $k\sigma$

$\mu = 10$
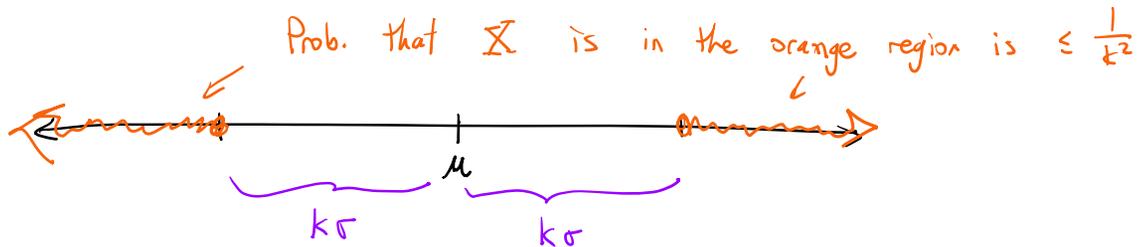
Example: The waiting time X for a bus has mean 10 minutes and standard deviation 2 minutes. Find $P(6 < X < 14)$.

$\sigma = 2$

$P\left(6 < X < 14\right) > \frac{3}{4}$

6   $\mu = 10$ 14

2σ 2σ

$k = 2$
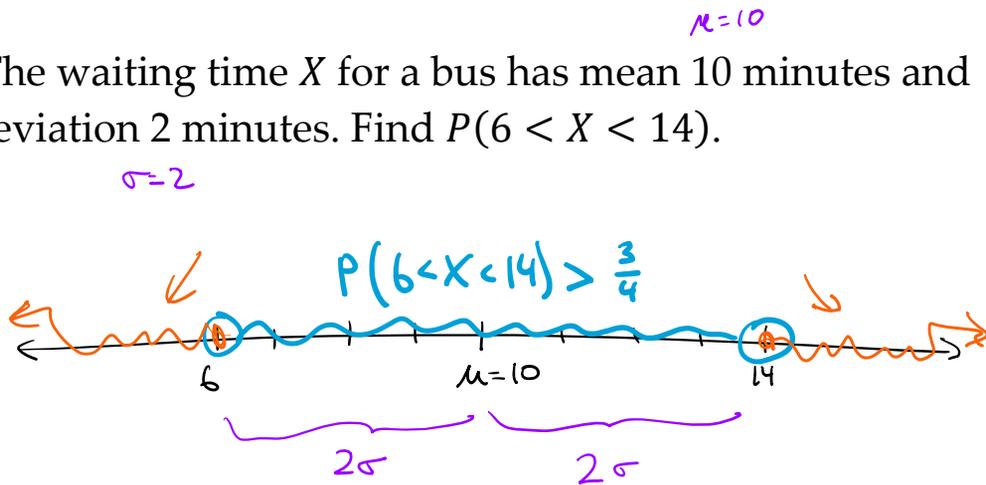
Chebyshev: $P\left(|X - \mu| \geq 2\sigma\right) \leq \frac{1}{2^2}$

$P\left(|X - 10| \geq 2(2)\right) \leq \frac{1}{4}$

orange region

$P\left(X \leq 6 \text{ or } X \geq 14\right) \leq \frac{1}{4}$

Therefore,

$P\left(6 < X < 14\right) > 1 - \frac{1}{4} = \frac{3}{4}$

Verify: $P\left(|X - \mu| \geq k\sigma\right) \leq \frac{1}{k^2}$ for $k = 2$

$P\left(|X - \mu| \geq 2\sigma\right) \leq \frac{1}{4}$

$P\left(|X - 6.6| \geq 2\sqrt{5.44}\right) \leq \frac{1}{4}$

$P\left(|X - 6.6| \geq 4.66\right) \leq \frac{1}{4}$

TRUE! $0 \qquad \leq \frac{1}{4}$