

**Warm-up:** A binomial experiment is characterized by what four properties?

1. Consists of  $n$  smaller experiments called trials.
2. Each trial results in "success" or "failure".
3. Trials are independent.
4. Probability of success  $p$  is same for each trial.

**Problem:** Let  $X \sim \text{Bin}(20, 0.6)$ . What is the probability that  $X$  is within 1.5 standard deviations of its mean?

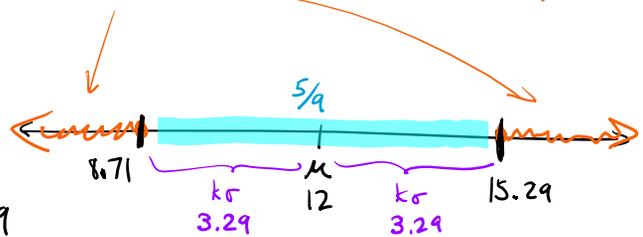
Chebyshev's Inequality:  $P(|X - \mu| \geq k\sigma) \leq \frac{1}{k^2}$        $P(|X - 12| \geq 3.29) \leq \frac{1}{1.5^2} = \frac{4}{9}$

choose  $k = 1.5$

Mean:  $\mu = np = 20(0.6) = 12$

st. dev:  $\sigma = \sqrt{np(1-p)} = \sqrt{20(0.6)(0.4)} = 2.19$

so  $k\sigma = 1.5(2.19) = 3.29$



complementary probability

$P(|X - 12| < 3.29) \geq 1 - \frac{4}{9} = \frac{5}{9} = 0.55\dots$

Exact Probability (since  $X$  is Binomial random variable)

$P(8.71 < X < 15.29) = P(X = 9 \text{ or } 10, \text{ or } 11, \dots, 15)$

$= \sum_{k=9}^{15} \binom{20}{k} (0.6)^k (0.4)^{20-k} = \underline{0.8925}$

Binomial  
 $P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$