MATH 262

Section 2.7 Day 16

1. Let $X \sim \text{Geometric}(p)$.

   (a) What is the mgf $M_X(t)$? Write your answer as a sum.

   (b) Your summation in part (a) should be a geometric series. Use the geometric sum formula

   $$a + ar + ar^2 + ar^3 + \cdots = \frac{a}{1-r}$$

   to write the mgf $M_X(t)$ *without* a sum.

2. Suppose random variable X has probability mass function $P(X = x) = \frac{27}{40}\left(\frac{1}{3}\right)^x$, for integers $0 \leq x \leq 3$.

   (a) Verify that this is a valid probability mass function.

   (b) Find the moment generating function $M_X(t)$ of X. Try to write the mgf without a sum.

(c) Compute $M'_X(0)$. (You can use technology!) Does your answer agree with the expected value of X computed directly from the pmf?

3. Let X be a discrete random variable with mgf $M_X(t)$, and let $Y = aX + b$. Write the definition of $M_Y(t)$. Replace Y with $aX + b$ and simplify the expected value to show that $M_Y(t) = e^{tb}M(at)$.

★ **BONUS:** Let $Y = aX+b$. Differentiate both sides of $M_Y(t) = e^{tb}M(at)$ to show that $E(Y) = aE(X)+b$. Differentiate again to show that $\text{Var}(Y) = a^2\text{Var}(X)$.