Moment Generating Functions:

$$M_X(t) = E(e^{tX}) = 1 + E(X)\,t + E(X^2)\,\frac{t^2}{2} + E(X^3)\,\frac{t^3}{3!} + \cdots$$

$$M_X(0) = 1$$

$E(X^0)\,t^0$

(1) To find $E(X^k)$ given $M_X(t)$: differentiate M_X k times and evaluate at $t=0$.

(2) If $Y = aX + b$, then $M_Y(t) = e^{bt} M_X(at)$.

why?

$$M_Y(t) = E(e^{tY}) = E\left(e^{t(aX+b)}\right) = E\left(e^{atX+bt}\right) = E\left(e^{bt}e^{atX}\right)$$

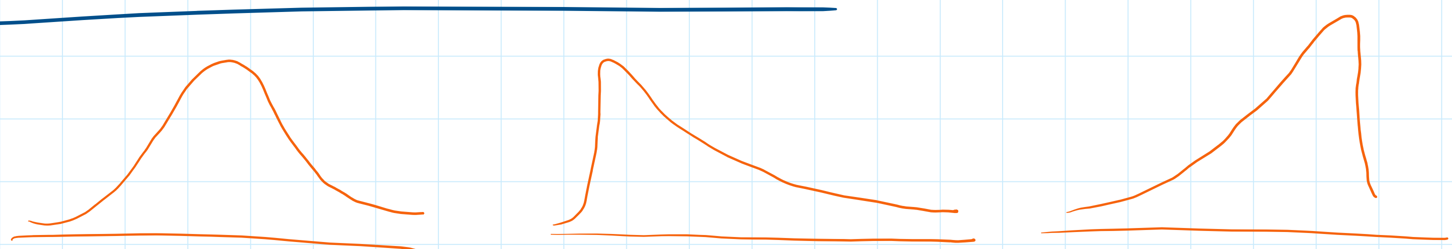
$$= e^{bt}E(e^{atX}) = e^{bt} M_X(at)$$

(3) The mgf uniquely specifies the distribution.

If two random variables have the same mgf, then they have the same distributions.

Uniqueness of mgfs

WORKSHEET PROBLEM 3: SKEWNESS



zero skewness — positive skewness — negative skewness

Skewness Coefficient:

$$\gamma = \frac{E[(X-\mu)^3]}{\sigma^3}$$

$$E[(X-\mu)^3] = E[X^3 - 3X^2\underset{\uparrow \atop E(X)}{\mu} + 3X\underset{\uparrow \atop E(X)}{\mu}^2 - \underset{\uparrow \atop E(X)}{\mu}^3]$$

$$= E(X^3) - 3E(X^2)E(X) + 2E(X)^3$$

$$\sigma = \sqrt{E(X^2) - E(X)^2}$$

$$\gamma = \frac{E(X^3) - 3E(X^2)E(X) + 2E(X)^3}{\left(E(X^2) - E(X)^2\right)^{3/2}}$$

- $X \sim Bin(10, \frac{1}{2})$ $M_X(t) = \left(1 - \frac{1}{2} + \frac{1}{2}e^t\right)^{10}$