DISCRETE SUMS ⟶ CONTINUOUS INTEGRALS

EXPECTED VALUE of X: $\mu = E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$

(value ↑, density ↑)

... of $h(X)$: $E(h(X)) = \int_{-\infty}^{\infty} h(x) \cdot f(x)\, dx$

VARIANCE of X: $Var(X) = E\left[(X-\mu)^2\right] = \int_{-\infty}^{\infty} (x-\mu)^2 \cdot f(x)\, dx$

$= E(X^2) - \left(E(X)\right)^2$

MOMENT GENERATING FUNCTION:

$$M_X(t) = E(e^{tX}) = \int_{-\infty}^{\infty} e^{tx} \cdot f(x)\, dx$$

(value ↑, density ↑)

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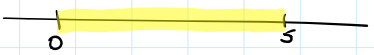
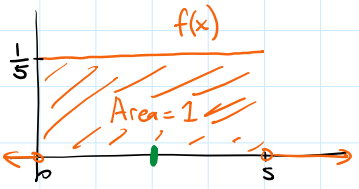
① $U \sim Unif[0, 5]$



ⓐ pdf:



Exp. Value: $E(U) = \int_0^5 u \cdot f(u)\, du = \int_0^5 u \cdot \frac{1}{5}\, du = \frac{1}{10}u^2 \Big|_0^5 = \frac{25}{10} - \frac{0}{10} = 2.5$

Var:

$$E(U^2) = \int_0^5 u^2 \cdot f(u)\, du = \int_0^5 \frac{1}{5}u^2\, du = \frac{1}{15}u^3 \Big|_0^5 = \frac{125}{15} - 0 = \frac{25}{3}$$

$$Var(U) = E(U^2) - E(U)^2 = \frac{25}{3} - \left(\frac{5}{2}\right)^2 = \frac{25}{3} - \frac{25}{4} = \frac{4(25) - 3(25)}{12} = \frac{25}{12}$$

If $X \sim Unif[A, B]$:

$E(X) = \frac{A+B}{2}$

$Var(X) = \frac{(B-A)^2}{12}$

⑥ Let $V = 3U + 2$. linear transformation

$E(V) = 3E(U) + 2 = 3(2.5) + 2 = 9.5$

$Var(V) = 3^2 \, Var(U) = 9 \cdot \frac{25}{12} = \frac{75}{4} = 18.75$

© What do you think is the distribution of V?

V should be $Unif[2, 17]$

② $M_U(t) = \begin{cases} 1 & \text{if } t = 0 \\ \frac{e^{5t} - e^{0t}}{5t} & \text{if } t \neq 0 \end{cases}$

$M_{aX+b}(t) = e^{bt} M_X(at)$

$V = 3U + 2$

$M_V(t) = e^{2t} M_U(3t) = e^{2t} \begin{cases} 1 & \text{if } t = 0 \\ \frac{e^{5(3t)} - e^{0}}{5(3t)} & \text{if } t \neq 0 \end{cases}$

$$M_V(t) = \begin{cases} 1 & \text{if } t = 0 \\ \frac{e^{17t} - e^{2t}}{15t} & \text{if } t \neq 0 \end{cases}$$

mgf of a $Unif[2, 17]$ distribution