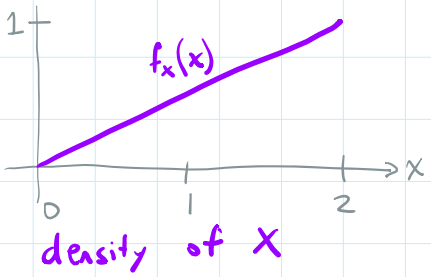
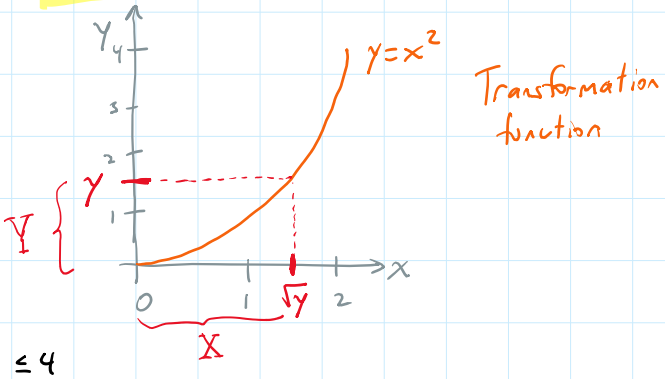
TRANSFORMATIONS OF RANDOM VARIABLES

1. Let X have density $f_X(x) = \frac{x}{2}$ for $0 \le x \le 2$, and let $Y = X^2$. What is the density of Y?



density of X



Note: Y takes values $0 \le y \le 4$

Find the cdf of Y: $F_Y(y) = P(Y \le y) = P(X^2 \le y) = P(X \le \sqrt{y})$

$$= \int_0^{\sqrt{y}} \frac{x}{2}\,dx = \frac{x^2}{4}\Big|_{x=0}^{x=\sqrt{y}} = \frac{(\sqrt{y})^2}{4} - \frac{0}{4} = \frac{y}{4}$$

$F_Y(y) = \frac{y}{4}$

Differentiate F_Y to obtain f_Y: $f_Y(y) = \frac{d}{dy}\left(\frac{y}{4}\right) = \frac{1}{4}$ for $0 \le y \le 4$

TRANSFORMATION THEOREM: If X has pdf $f_X(x)$ and $Y = g(X)$ where g is monotonic (strictly increasing or decreasing), then:

$$f_Y(y) = f_X(h(y)) \cdot |h'(y)|$$

where $h(y)$ is the inverse function of $g(x)$.

In Problem #1: $g(x) = x^2$, so $h(y) = \sqrt{y}$

Then: $f_Y(y) = f_X(\sqrt{y}) \cdot \left|\frac{1}{2\sqrt{y}}\right| = \frac{\sqrt{y}}{2} \cdot \frac{1}{2\sqrt{y}} = \frac{1}{4}$