

MATH 262

Section 4.5

Day 33

1. Simulate 10,000 averages, each of k samples from a $\text{Unif}[0, 1]$ distribution. Make a histogram of the 10,000 averages. Start with $k = 1$ and then try larger values of k . How does the shape of the histogram depend on k ?

Here is some code for making such a histogram in *Mathematica*:

```
averages = Table[ Mean[ RandomVariate[ UniformDistribution[], k ] ], 10000 ]  
Histogram[averages]
```

Here is similar code in R:

```
averages <- replicate( 10000, mean(runif(k)) )  
hist(averages)
```

Sketch the shape of your histograms:

2. Repeat the previous simulation, but now replace $\text{Unif}[0, 1]$ with a different distribution of your choice. What is the shape of the histogram? How does it depend on k ?

3. Let X_1, X_2, \dots, X_{300} be iid random variables with mean μ_X and standard deviation σ_X . Also let $T = X_1 + X_2 + \dots + X_{300}$ and $\bar{X} = \frac{T}{300}$.

(a) What are the values of μ_T , σ_T , $\mu_{\bar{X}}$, and $\sigma_{\bar{X}}$?

(b) What distributions are good approximations for T and \bar{X} ?

4. Use the `Convolve` function in Mathematica to plot the pdf of $X_1 + X_2 + \dots + X_n$, where each $X_i \sim \text{Unif}[0, 1]$ and $n \in \{1, 2, 3, 4, 5, 6\}$. Compare each pdf with the pdf of a normal distribution.

5. A farm packs tomatoes in crates. Individual tomatoes have mean weight of 10 ounces and standard deviation of 3 ounces. Estimate the probability that a crate of 40 tomatoes weighs between 380 and 410 ounces.