

# MATH 262

Section 4.6

Day 36

1. Suppose  $X_1$  and  $X_2$  are iid  $\text{Unif}[0, 1]$ . Let  $Y_1 = X_1 + X_2$  and  $Y_2 = X_1 - X_2$ .

(a) *Discuss with your neighbors:* What is the region of possible values of the pair  $(Y_1, Y_2)$ ? (This is a region in the  $y_1y_2$ -plane.)

(b) Find the inverse transformation functions  $v_1$  and  $v_2$  such that  $X_1 = v_1(Y_1, Y_2)$  and  $X_2 = v_2(Y_1, Y_2)$ .

(c) Use the transformation theorem to find the joint pdf of  $Y_1$  and  $Y_2$ .

2. Let  $X_1$  and  $X_2$  have joint density  $f(x_1, x_2) = \frac{1}{x_1^2 x_2^2}$  for  $x_1 \geq 1$  and  $x_2 \geq 1$ . Let  $Y_1 = X_1 X_2$  and  $Y_2 = \frac{X_1}{X_2}$ .

(a) Show that the region of positive joint density for  $Y_1$  and  $Y_2$  is given by  $1 \leq Y_1$  and  $\frac{1}{Y_1} \leq Y_2 \leq Y_1$ .

(b) Find the joint pdf of  $Y_1$  and  $Y_2$ .

3. Let  $(X, Y)$  be a random point in the plane, where  $X$  and  $Y$  are independent standard normal random variables. Let  $(R, \Theta)$  be the polar coordinates of  $(X, Y)$ . Find the joint density of  $R$  and  $\Theta$ . Then find the marginal densities of  $R$  and  $\Theta$ . What is the probability that  $(X, Y)$  lies inside a circle of radius 1 centered at the origin?