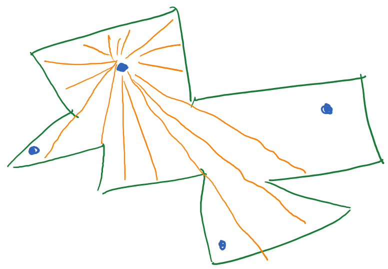
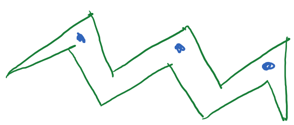
Suppose an art gallery is modeled by a polygon.
A guard occupies a single point and can see in all directions to the walls.

example:

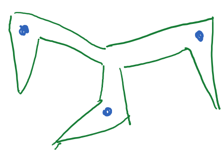


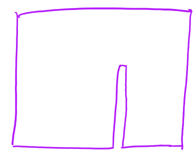
How many guards are necessary to cover an n-sided polygon?

$\lceil \frac{n-2}{4} \rceil$



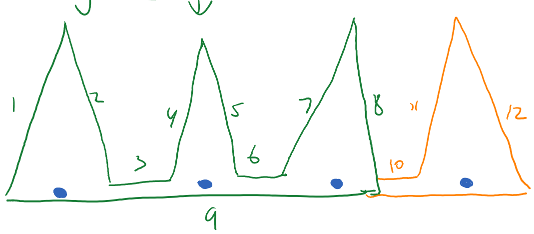
$1+\lfloor \frac{n-2}{4} \rfloor$





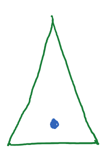
interior wall

9 edges
3 guards

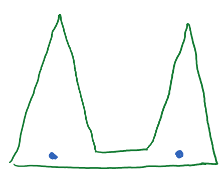


CONJECTURE: To cover a polygon with n vertices, $\lfloor \frac{n}{3} \rfloor$ guards are necessary for some polygons and sufficient for all polygons.

Proof of necessary:



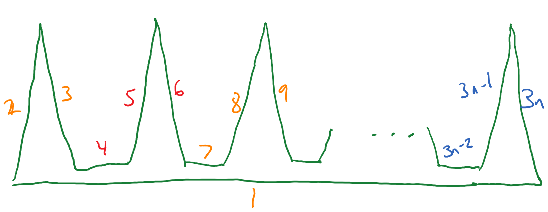
$n=3$, requires 1 guard



$n=6$

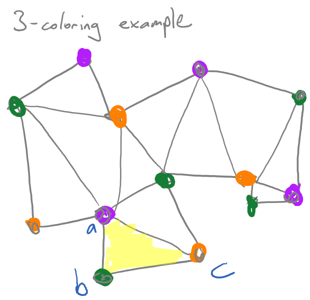
requires 2 guards

in general:



This polygon is like a comb with n "teeth."

Proof of sufficient: Let P have n vertices.
Triangulate P.



The vertices of P may be 3-colored such that any 2 vertices connected by an edge or a diagonal have different colors.

base case: If $n=3$, then P is a triangle, so color each vertex a different color.

induction: Assume true for polygons with $n \geq 3$ vertices.
Suppose P has $n+1$ vertices.
Since P has at least 4 vertices, P has an ear.
Removing the ear (a, b, c) produces a polygon Q with n vertices.
By the induction hypothesis, Q can be 3-colored.
Replace the ear and color vertex b differently from a and c.

Placing guards on all vertices of any one color is guaranteed to cover the polygon.

The least frequently used color occurs at most $\left\lfloor \frac{n}{3} \right\rfloor$ times.

Placing guards on these vertices is sufficient.