Take-Home Quiz 1
Math 282 Computational Geometry
Spring 2019

This quiz is like a regular homework assignment, except that:

- You must work by yourself. No collaboration or discussion with other students.

- You may only ask questions of Prof. Wright.

You may still use your textbook and computer. Submit your solutions (on Moodle or in the homework box) by 4:00pm on **Friday, March 8**.

If you are taking this course for elective credit towards the computer science major, then do the problem labeled CS only and not the problems labeled math only. Otherwise, do one of these two problems.

1. If P is a convex decagon (10 sides), in how many ways can it be triangulated?

2. Let B_1 and B_2 be two rectangular boxes of the same volume, with different base rectangles but the same height $h > 0$. Is there always a dissection (into a finite number of polyhedra) of B_1 to B_2. Select one of the following answers and defend your choice:

   (a) No, there is never such a dissection.
   (b) Yes, there is always such a dissection, for any h.
   (c) There is a dissection for some values of h but not for all values of h.

3. Let P be a convex polyhedron whose surface is triangulated. Is it true that every tetrahedralization of P results in the same number of tetrahedra?

4. Define a *spiral polygon* to be composed of a sequence of convex angles followed by a sequence of one or more reflex angles. Find an art-gallery theorem for such polygons with n total vertices. How many guards are sometimes necessary, and always sufficient, to completely cover the interior of a spiral polygon with n vertices, as a function of n?

   Hint: Find a "worst" case that establishes necessity, and then try to find an argument that establishes sufficiency. You might find it convenient to think in terms of the number c of convex vertices and the number r of reflex vertices.

5. Below is an 8×8 square dissected to a 13×5 rectangle. Something is not right. Find and explain the flaw.
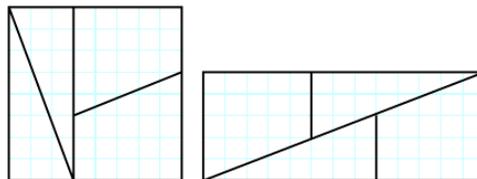
6. math only: Let the *diameter* of a point set S be the largest distance between any two points of S. Prove that the diameter of S is achieved by two vertices in conv(S).

7. CS only: Implement the Graham Scan convex hull algorithm in your favorite programming language. Provide sample input and output to demonstrate that your algorithm works.