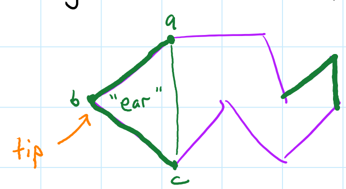
Last Time: If polygon P has n vertices, then every triangulation of P has n-2 triangles.

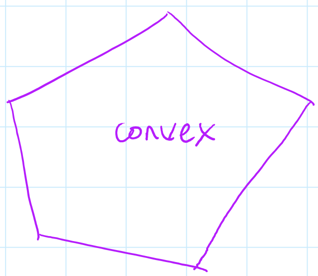
Today: How many triangulations does a polygon have?

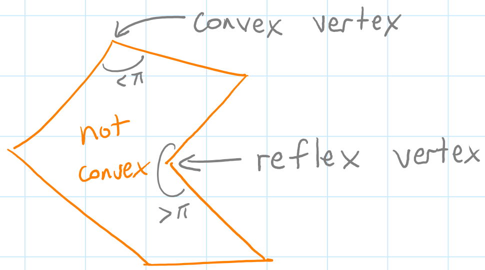
DEF: If a, b, c are consecutive vertices of a polygon and a, c is a diagonal, then a, b, c is called an **EAR**.



Cor. 1.9: Every polygon with more than 3 vertices has at least two ears with non-adjacent tips.

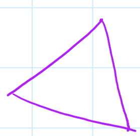
CONVEX POLYGON: A polygon such that the line connecting any two vertices is inside the polygon.





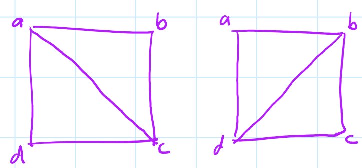
QUESTION: If P is a convex polygon with n vertices, how many triangulations does P have?

n = 3



1 triangulation

n = 4

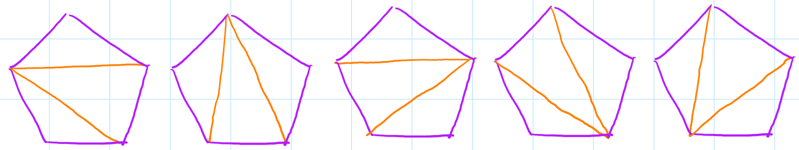


2 triangulations

n = 5 ?

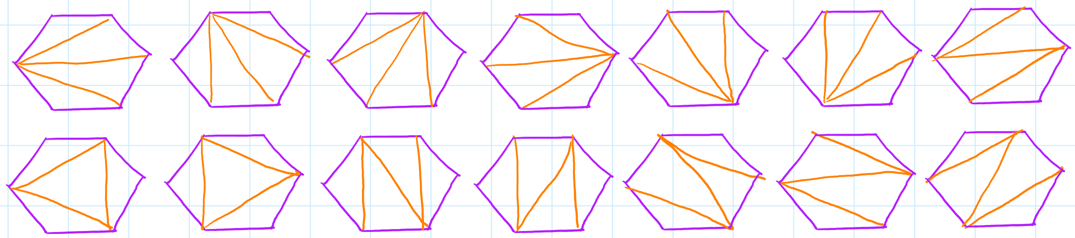
n = 6 ?

$n=5$



5 triangulations

$n=6$



14 triangulations

THEOREM: If P is a convex polygon with $n+2$ vertices, then the number of triangulations is the Catalan number

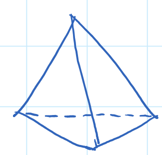
$$C_n = \frac{1}{n+1}\binom{2n}{n}. \qquad \binom{2n}{n} = \frac{(2n)!}{n!\,n!}$$

Check: 3 vertices $\longrightarrow$ $n=1$: $C_1 = \frac{1}{1+1}\binom{2}{1} = \frac{1}{2}\cdot\frac{2!}{1!\,1!} = \frac{2}{2} = 1$

4 vertices $\longrightarrow$ $n=2$: $C_2 = \frac{1}{2+1}\binom{4}{2} = \frac{1}{3}\cdot\frac{4!}{2!\,2!} = \frac{1}{3}\cdot\frac{24}{2\cdot 2} = 2$

5 vertices $\longrightarrow$ $n=3$: $C_3 = \frac{1}{3+1}\binom{6}{3} = \frac{1}{4}\cdot\frac{6!}{3!\,3!} = \frac{1}{4}\cdot\frac{720}{6\cdot 6} = 5$

POLYHEDRA: tetrahedralization — partion the polyhedra into tetrahedra



- Not every polyhedra has a tetrahedralization.
  example: Schönhardt polyhedron
- Two tetrahedralizations of the same polyhedra might have different numbers of tetrahedra.

example: