

# LOWER BOUND OF COMPLEXITY FOR 2D CONVEX HULL ALGORITHMS

Convex hull of  $n$  points in a plane cannot be found faster than  $O(n \log n)$  time.

Notation:  $\Omega(n \log n)$  is the lower bound for complexity of hull algorithm.

The same lower bound holds if we just want to identify hull points, even without putting them in order.

## DIVIDE AND CONQUER ALGORITHM

Recursive algorithm: algorithm that calls itself  
(similar to proof by induction)

**PSEUDOCODE:** Sort the points by x-coordinate

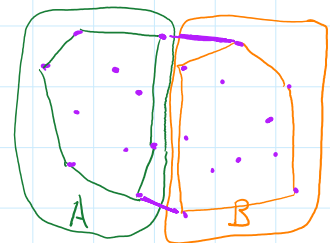
A = left half of the points

B = right half of the points

find convex hull of A

find convex hull of B

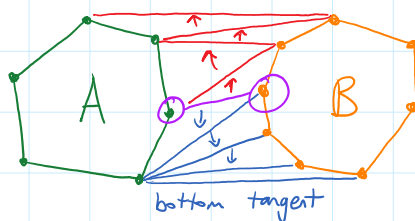
This is the complicated part → merge the two hulls together



how to do this?  
use this alg!

### MERGING TWO HULLS:

$O(n)$   
complexity



Identify the rightmost point of A and the leftmost point of B.

"Walk" the edge down until all right turns are eliminated. Similarly, "walk" the edge up.

## OVERALL COMPLEXITY OF DIVIDE-AND-CONQUER

Sort points:  $O(n \log n)$

Divide points in half  $\log_2(n)$  times  
Merge back together takes  $O(n)$  time }  $O(n \log n)$

Overall complexity:  $O(n \log n)$