CONVEX HULLS IN 3D

For n points in 3D, the convex hull has at most 3n edges and 2n faces.

linear in n

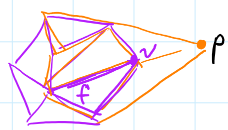
INCREMENTAL ALGORITHM

$O(n^2)$, but easy to implement

Start with four points; hull is a tetrahedron.

Add new points one by one. — Need to find which part of the hull is visible from a new point.

Suffices to determine which faces are visible from p.



IDEA: Face f is visible from point p if and only if $N \cdot (v-p) < 0$, where N is a vector normal to f and v is any vertex of f.

DIVIDE AND CONQUER ALGORITHM

$O(n \log n)$ but merging two hulls is complicated in 3D

Merging can be accomplished in $O(n)$ time by a "wrapping" algorithm.

Deleting interior edges / faces is nontrivial — see O'Rourke and Edelsbrunner example

GRAHAM SCAN: No known 3D version!

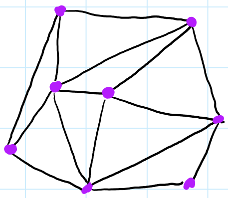
GIFT WRAPPING: $O(nf)$, where f is number of faces in hull

CHAPTER 3

TRIANGULATIONS OF POINTS IN THE PLANE

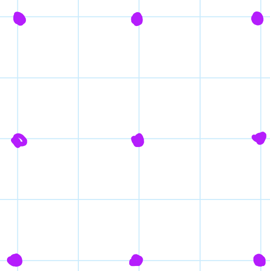
A **TRIANGULATION** of a set of points S in the plane is a subdivision of the plane determined by a maximal set of noncrossing edges whose vertex set is S.

EXAMPLE:



S = set of purple points

PROBLEM: Triangulate the following 3x3 lattice.



How many triangles in a triangulation?

How many triangulations are possible?