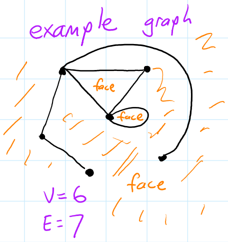
EULER'S FORMULA:



$V=6$
$E=7$
$F=3$

Let G be a connected planar graph with $V \geq 1$ vertices, E edges, and F faces (including the unbounded outer face).

Then: $V - E + F = 2$

proof: Induction on the number of edges.

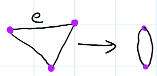
BASE CASE: If $E=0$, then $V=1$ since graph is connected and $F=1$. So: $V-E+F = 1-0+1 = 2$

INDUCTION: Assume the formula holds for graphs with up to $E-1$ edges.

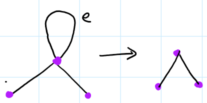
Suppose graph G has E edges.

Choose any edge e of G. There are two options:

(1) Edge e connects two different vertices.
Contract e, reducing V and E by 1.
This does not change the sum $V-E+F$.

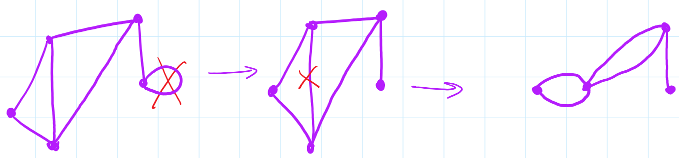


(2) Edge e forms a loop.
Delete edge e, reduces E and F by 1.
Again, this does not change $V-E+F$.



In either case, we reduce G to a graph with $E-1$ edges, without changing the sum $V-E+F$.

EXAMPLE:



$V=5$, $E=7$, $F=4$ — $5-7+4=2$

$V=5$, $E=6$, $F=3$ — $5-6+3=2$

$V=4$, $E=5$, $F=3$ — $4-5+3=2$

TRIANGULATIONS OF POINT SETS

THEOREM: Let S have h points in its convex hull and k interior points. (Assume not all points are collinear.)

Then: any triangulation of S has exactly $2k+h-2$ triangles and $3k+2h-3$ edges.

proof: Let $n = h+k$, the total number of points.
Let t be the number of triangles in a triangulation of S.

Then: $V = n$

$F = t+1$, including the outer face

$E = \dfrac{3t+h}{2}$ ← numerator counts all edges twice

Euler's Formula: $V - E + F = n - \dfrac{3t+h}{2} + t + 1 = 2$

$$2n - 3t - h + 2t + 2 = 4$$

$$2n - t - h = 2$$

solve for t:

$$t = 2n - h - 2$$

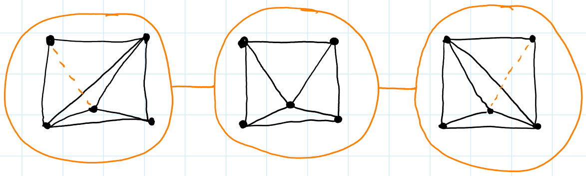
$$t = 2(h+k) - h - 2$$

$$\boxed{t = 2k + h - 2}$$ number of triangles

number of edges: $E = \dfrac{3t+h}{2} = \dfrac{3(2k+h-2)+h}{2} = \dfrac{6k+4h-6}{2} = \boxed{3k+2h-3}$

FROM ONE TRIANGULATION TO ANOTHER

example:



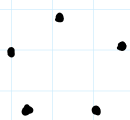
FLIP GRAPH

Vertices: triangulations

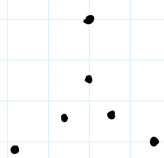
edges: connect two triangulations that differ by a single edge flip

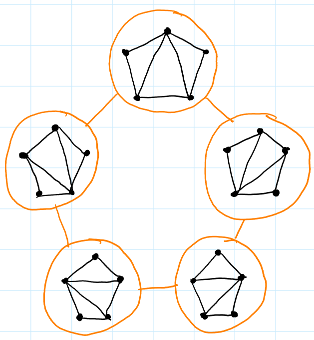
PROBLEMS: Draw the flip graph for each set of points

(a)



(b)





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QUESTION: Is the flip graph always connected?