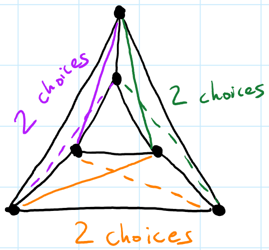
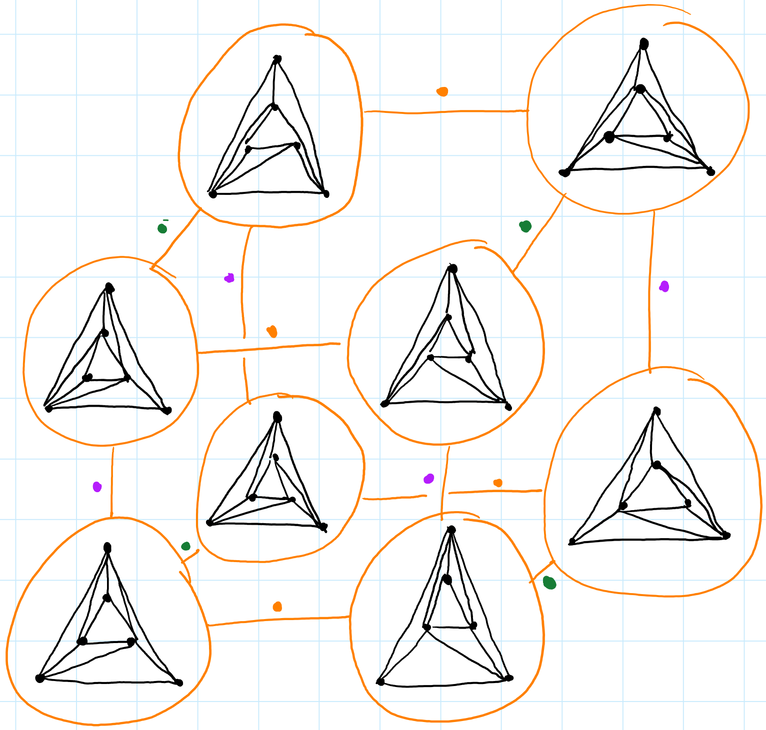
From last time:





THEOREM: The flip graph of any point set in the plane is connected.

Thus, any triangulation can be transformed into any other triangulation by edge flips.

Proof: Order points by x-coordinate, and let T_* be the triangulation obtained by the incremental algorithm.

Claim: Any other triangulation can be transformed into T_* by flips.

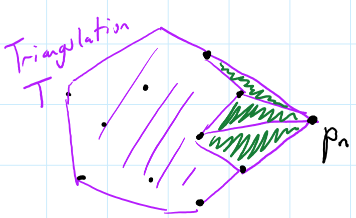
Proof by induction on the number of vertices n.

BASE CASE: $n=3$, the there is only one triangulation — Done.

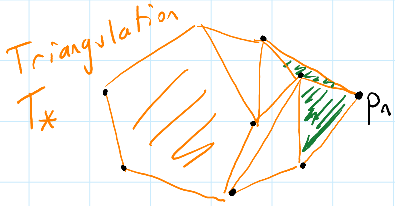
INDUCTION: Assume true for less than n points.

$S = \{p_1, \ldots, p_n\}$ with triangulation T.

Consider the star of p_n, which is the set of triangles adjacent to p_n.

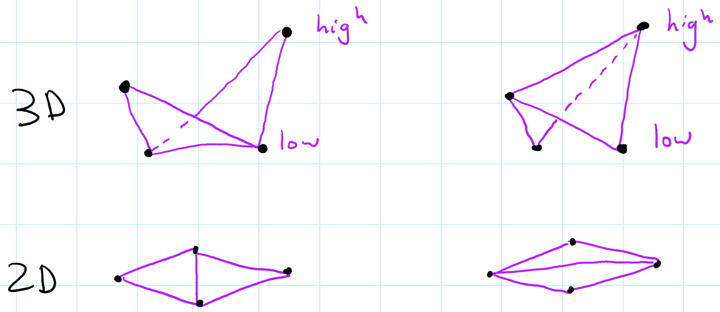


In T, repeatedly flip edges connecting p_n to other vertices, until p_n is only connected to vertices visible to p_n on the convex hull of the other $n-1$ vertices.



This reduces the problem to the $n-1$ case.

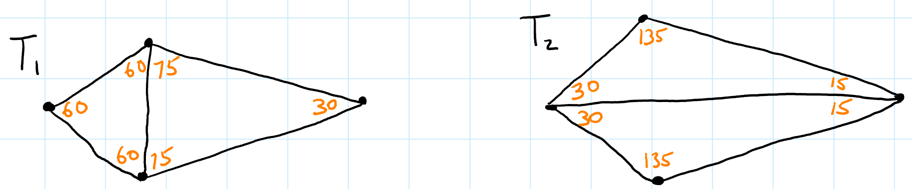
Example:



Different triangulations of the same points can have different features

DEFINITION: Triangulation T_1 is "fatter" than T_2 if the sorted list of angles of T_1 is lexicographically larger than that of T_2.

example:



angles: 30, 60, 60, 60, 75, 75   15, 15, 30, 30, 135, 135

30 > 15

So, T_1 is "fatter" than T_2, denoted $T_1 > T_2$

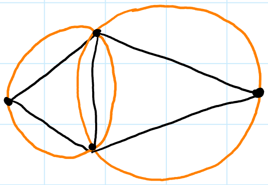
lexicographically: apple, apricot, book, bill

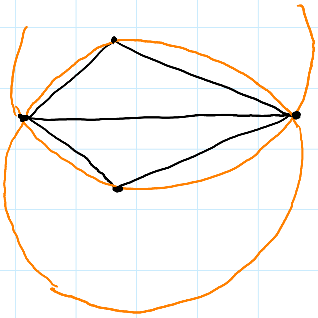
Dictionary Order

The **DELAUNAY TRIANGULATION** is the fattest triangulation.

THEOREM: A triangulation T is a Delaunay triangulation if no point of S is inside the circumcircle of any triangle in T.

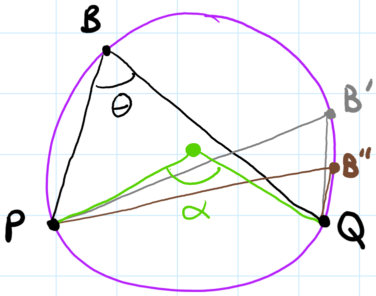
example:





QUESTION:

Points P, Q fixed on a circle



How does angle θ depend on the position of B?

θ stays the same!

$\theta = \frac{\alpha}{2}$