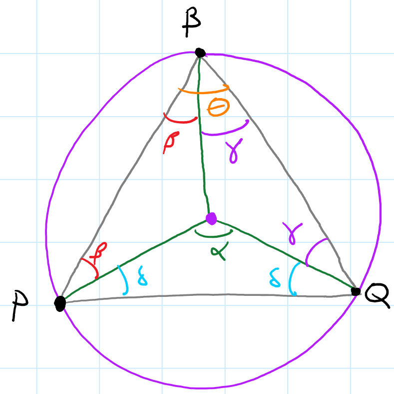
INSCRIBED ANGLE THEOREM:



Proof that $2\theta = \alpha$

Note: $2\beta + 2\gamma + 2\delta = 180^\circ$

Subtract: $-\left(\alpha + 2\delta = 180^\circ\right)$

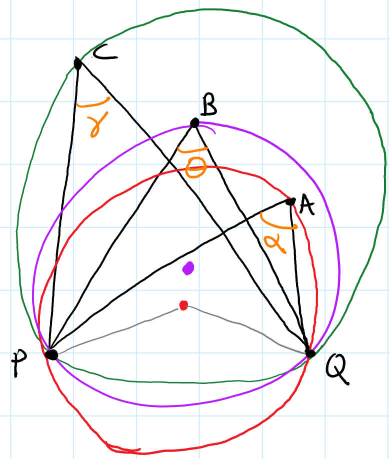
$$2\beta + 2\gamma - \alpha = 0$$

Thus, $2(\beta + \gamma) = \alpha$. Since $\theta = \beta + \gamma$, we have $2\theta = \alpha$.

THALES' THEOREM:

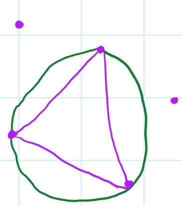
For points as in the diagram:

$$\alpha > \theta > \gamma$$



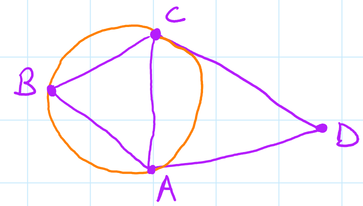
LAST TIME: We <u>defined</u> the Delaunay triangulation of a point set S to be the triangulation with the maximum list of angles (in lexicographical order).

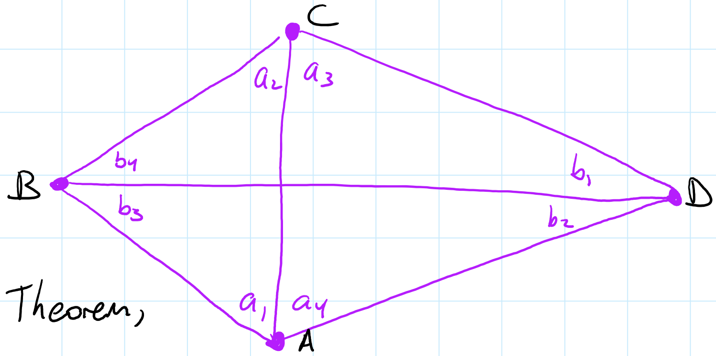
We said that no point of S in interior to the circumcircle of any triangle in the Delaunay triangulation.



PROPOSITION: Let $e = AC$ be an edge of triangles ABC and ACD in a triangulation.

Then e is an edge of the Delaunay triangulation if and only if D is outside of the circumcircle of ABC.





By Thales' Theorem,

$b_1 < a_1$, $b_2 < a_2$, $b_3 < a_3$, $b_4 < a_4$

If edge AC is part of the triangulation, then the triangulation has angles $\underbrace{a_1, a_2, a_3, a_4, b_3+b_4, b_1+b_2}$

If edge BC is part of the triangulation, then we have angles $\underbrace{b_1, b_2, b_3, b_4, a_2+a_3, a_1+a_4}$

one of these is the smallest angle

Thus, the sequence of angles for the triangulation involving angle AC is larger lexicographically.

Edge AC is part of the Delaunay Triangulation.