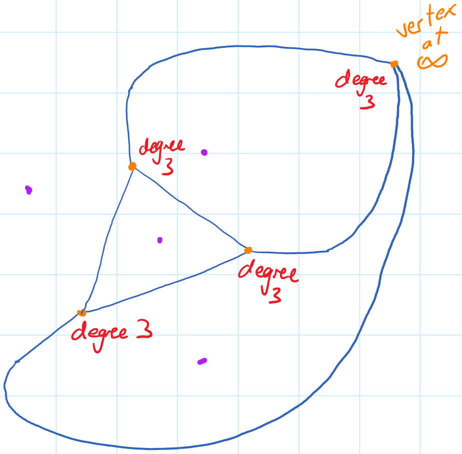
THEOREM: Let S be a point set with $n \geq 3$ sites. Then $\mathrm{Vor}(S)$ has at most $2n-5$ Voronoi vertices and $3n-6$ Voronoi edges.



proof: Add a new vertex "at ∞" and bend all the ∞ edges to this vertex. This creates a planar graph G.

G has: n faces

e = number of edges in $\mathrm{Vor}(S)$

$v+1$ vertices, where v = number of vertices in $\mathrm{Vor}(S)$

Euler's Formula: $n - e + (v+1) = 2$

Note: sum the degrees of the vertices of G counts each edge twice. Also, each Voronoi vertex has degree ≥ 3, so sum of degrees is at least $3(v+1)$.

Thus:
$$2e \geq 3(v+1)$$

First: Solve Euler's formula for $v+1$: $v+1 = 2-n+e$

Then:
$$2e \geq 3(2-n+e)$$
$$2e \geq 6-3n+3e$$
$$\boxed{3n-6 \geq e}$$
so number of edges is at most $3n-6$

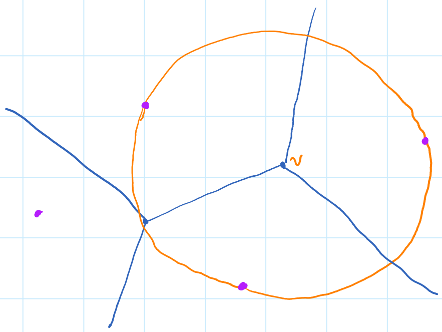
Second: Solve Euler's formula for e: $e = (v+1)+n-2$

Then:
$$2(v+1+n-2) \geq 3(v+1)$$
$$2v+2+2n-4 \geq 3v+3$$
$$\boxed{2n-5 \geq v}$$
so the number of vertices is at most $2n-5$

THEOREM 4.6: A point v is a Voronoi vertex iff and only if there is a circle centered at v with 3 or more sites on its boundary and none in its interior.

(proof: in book)

example:



The dual graph of the Voronoi diagram <u>is</u> the Delaunay Triangulation!

<u>why?</u>

THEOREM: If S is a point set with no four sites cocircular, then the dual graph of $\mathrm{Vor}(S)$ is the Delaunay triangulation of S.

<u>proof:</u> By Theorem 4.6, the circumcircle of each triangle in the dual graph of $\mathrm{Vor}(S)$ has no sites in its interior.

Also, since no four points are cocircular, the circumcircle of any triangle in the dual graph contains only the vertices of that triangle.

Thus, the triangulation given by the dual graph of $\mathrm{Vor}(S)$ has the $\underbrace{\text{"empty circle property"}}_{\text{Theorem 3.53 in the text}}$ that characterizes the Delaunay triangulation.