Why is the lower convex hull of sites projected onto the paraboloid equivalent to the Delaunay triangulation?

Choose a face T of the lower convex hull.

Let π be the plane containing T.

So π is determined by the 3 vertices of T on the paraboloid.

The intersection of π with the paraboloid is an ellipse, which projects to a circle C in the xy-plane.

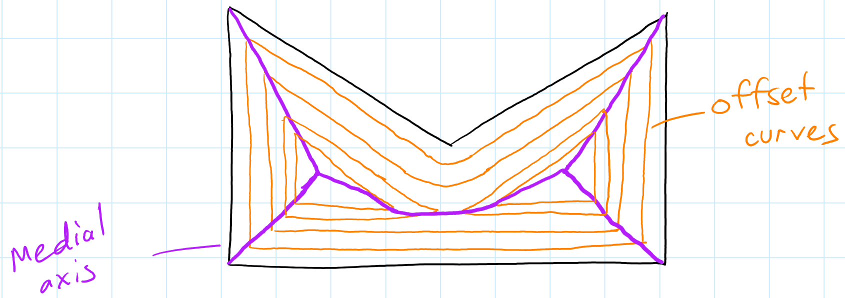
contains exactly 3 sites in the xy-plane

Since π is a face of the lower hull, all other sites on the paraboloid lie above π, and so are outside of the intersection ellipse. Thus, all other sites project outside of the circle C in the xy-plane.

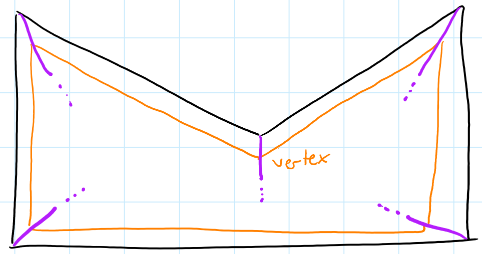
Thus, circle C contains exactly 3 sites on its boundary and none in its interior, so it is part of the Delaunay triangulation.

The same is true for all other triangles in the convex hull.

MEDIAL AXIS



STRAIGHT SKELETON



to be continued...