QUESTION: Let A and B be convex polygons with m and n vertices, respectively. What can we say about the Minkowski sum $A \oplus B$?

- What properties does $A \oplus B$ have? convex?
- How many edges does $A \oplus B$ have? up to $m+n$?
- What algorithm could construct $A \oplus B$?

DEFINITION: The **MINKOWSKI SUM** of sets A and B is

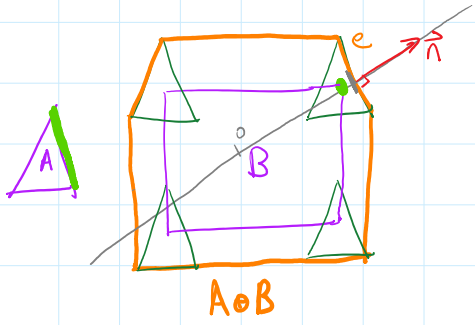
$$A \oplus B = \{ x + y \mid x \in A,\ y \in B \}$$

where $x + y$ is the vector sum of $x \in A$, $y \in B$.

PROPERTY: If A and B are convex, then so is $A \oplus B$.

PROPERTY: $A \oplus B$ has at most $m+n$ edges, and exactly $m+n$ edges if A and B have no parallel edges.

why? Let e be an edge of $A \oplus B$.



Edge e is extreme in the direction of its outer normal vector $\vec{n}$.

Edge e must be generated by points on A and B thare are also extreme in the direction of n.

If A and B have no parallel edges, then each edge of $A \oplus B$ arises from only one edge of A or B.

Thus, $A \oplus B$ has $m+n$ edges if A and B have no parallel edges. (No edge of A is parallel to any edge of B.)

What if A and B are not convex?

A has m edges	B has n edges	number of edges in $A \oplus B$
convex	convex	$O(m+n)$
convex	not convex	$O(mn)$
not convex	not convex	$O(m^2n^2)$