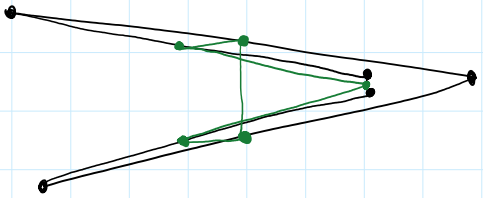
QUESTION: Does the midpoint transformation on a polygon ever produce a non-polygon?



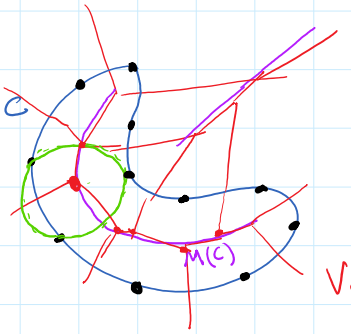
APPLICATION: CURVE RECONSTRUCTION

CRUST Algorithm: relates to medial axis, Voronoi diagram, and Delaunay transformation

(Nina Amenta, Marshall Bern, David Eppstein, 1998)

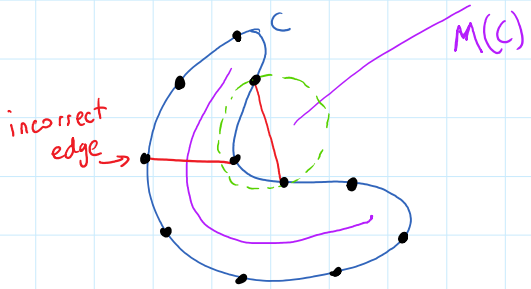
INSIGHTS:

1. Voronoi vertices of Vor(S) lie near the medial axis of C.
   - S: sample set of points
   - C: actual curve
   - M(C)



Voronoi vertices are equidistant from 3 sample points.

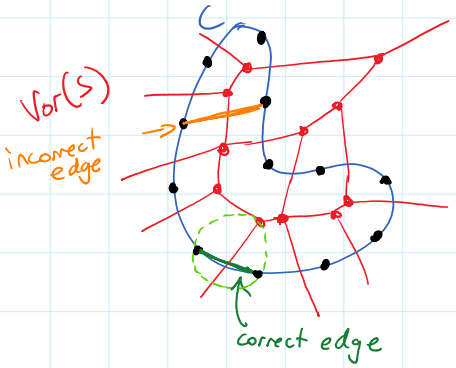
2. (Any?) circumscribing disk of an incorrect edge of Del(S) crosses the medial axis M(C).
   - incorrect edge: an edge that does not approximate curve C; an edge that connects two sample points that are not consecutive on C.



Correct edges are "far" from M(C).
Incorrect edges are "close" to M(C).

3. An incorrect edge e of $Del(S)$ cannot appear in $Del(S \cup V)$ because a circumscribing disk contains a point of V.

↑ Voronoi vertices of $Vor(S)$

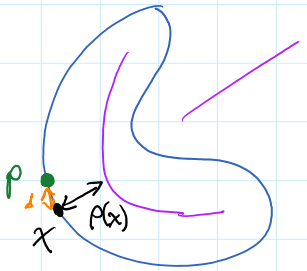


Follows from the previous two insights.

4. Each correct edge of $Del(S)$ also appears in $Del(S \cup V)$.

PROVABLE CORRECTNESS:

For a smooth curve C, and x a point on C, the **LOCAL FEATURE SIZE** $\rho(x)$ is the shortest distance from x to the medial axis $M(C)$.



Let $\varepsilon \in (0,1)$. A set of points S sampled from C is an **ε-SAMPLE** if each point $x \in C$ has a point $p \in S$ such that $|x-p| \leq \varepsilon \rho(x)$.

$d = |p-x| \leq \varepsilon \rho(x)$

THEOREM: The CRUST algorithm outputs the correct polygonal reconstruction whenever S is an ε-sample with $\varepsilon < \frac{1}{5}$.