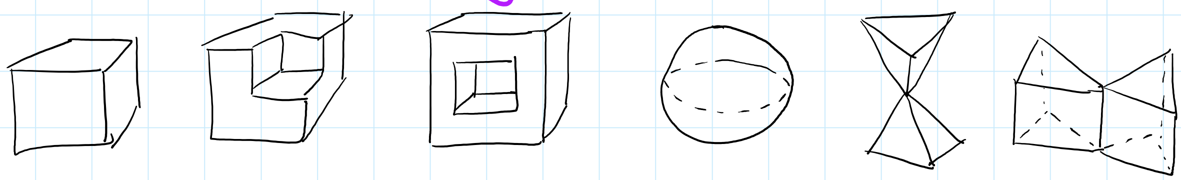
Which of the following are polyhedra?

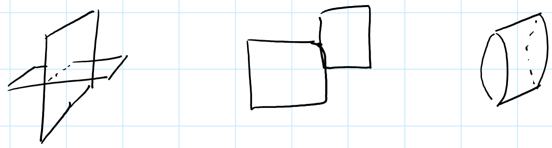


DEFINITION OF POLYHEDRON

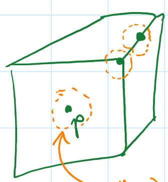
A polyhedron is composed of polygonal faces and satisfies the following requirements:

1. **INTERSECTION CONDITION:** Any two faces may only intersect along a single edge or at a single vertex.

Not Allowed:



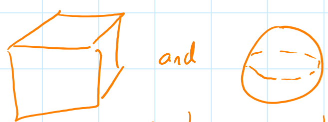
2. **LOCAL TOPOLOGY:** Every point p on the surface of a polyhedron has a neighborhood homeomorphic to an open disk.



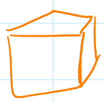
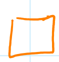
neighborhood of p is a set of all points on the polyhedron within some small distance of p

Two surfaces are homeomorphic if they can be continuously deformed into each oth (without gluing, tearing, or collapsing dimension).

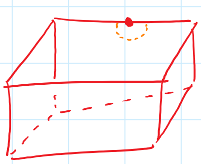
EXAMPLES:



are homeomorphic

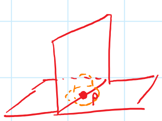
 and  are not homeomorphic

Not allowed:



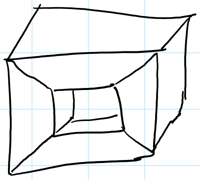
open box (no top)

 not homeomorphic to 

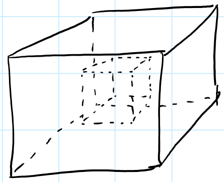


 not homeomorphic to 

3. GLOBAL TOPOLOGY: Surface of polyhedron is connected.



OK



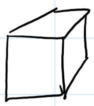
not allowed: cube with a floating cavity

EXERCISE: For a polyhedron on the table in front of you, compute $V - E + F$.

number of vertices − edges + faces

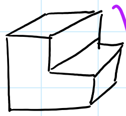
NOTATION: for a polyhedron P,

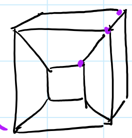
$$\underbrace{\chi(P)}_{\text{EULER CHARACTERISTIC}} = V - E + F$$

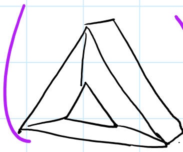
$\chi($  $) = 8 - 12 + 6 = 2$

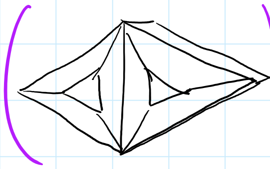
$\chi(\text{dodecahedron}) = 20 - 30 + 12 = 2$

$\chi($  $) = 4 - 6 + 4 = 2$

$\chi($  $) = 2$

$\chi($  $) = 12 - 24 + 12 = 0$

$\chi($  $) = 9 - 18 + 9 = 0$

$\chi($  $) = 14 - 32 + 16 = -2$

THEOREM: If polyhedron P is homeomorphic to a sphere, then $\chi(P) = 2$.

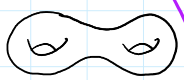
NOTE: Euler characteristic applies not just to polyhedra, but to surfaces (and lots of other objects).

EXAMPLES:

$\chi($  $) = 2$

$\chi($  $) = 0$

Euler characteristic is invariant under homeomorphism.

$\chi\Big($  $\Big) = -2$

THEOREM: For a (closed, orientable) surface S with g holes, the Euler characteristic $\chi(S) = 2 - 2g$.

DEF: The **GENUS** of a surface is the number of holes (i.e. tunnels).

QUESTION: If every face of a polyhedron is either a pentagon or a hexagon, and all vertices have degree 3, then how many faces are pentagons?

Total degree: $3V = 2E$

Let n be the number of pentagons, and m be the number of hexagons.

Then: $F = m + n$

$$V = \frac{5n+6m}{3}$$

$$5n + 6m = 3V = 2E$$

$$E = \frac{5n+6m}{2}$$

Euler characteristic: $V - E + F = 2$

$$6\left(\frac{5n+6m}{3} - \frac{5n+6m}{2} + (m+n)\right) = (2)6$$

$$\underline{10n} + \underline{12m} - \underline{15n} - \underline{18m} + \underline{6m} + \underline{6n} = 12$$

$$n = 12$$