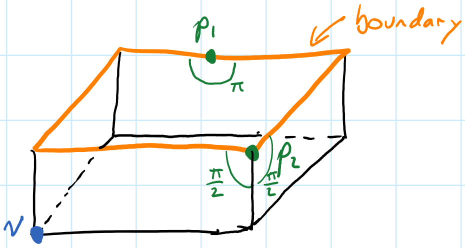
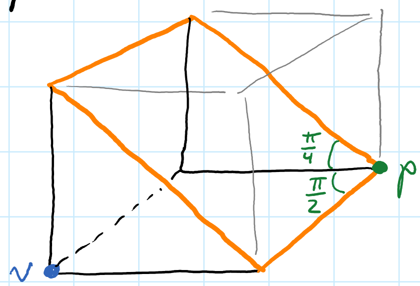
CURVATURE FOR POLYHEDRA WITH BOUNDARY

Let P be a polyhedron with boundary. The **GEODESIC CURVATURE** $K(p)$ at a boundary point p of P is π minus the sum of the face angles at p.

EXAMPLES:





curvature: $K(p_1) = \pi - \pi = 0$

$K(p_2) = \pi - 2\left(\frac{\pi}{2}\right) = 0$

$K(p) = \pi - \left(\frac{\pi}{4} + \frac{\pi}{2}\right) = \frac{\pi}{4}$

from last time: $K(v) = 2\pi - 3\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$

$K(v) = \frac{\pi}{2}$

Add up curvature at all vertices, including boundary vertices:

Total curvature: $4(0) + 4\left(\frac{\pi}{2}\right) = 2\pi$

$4\left(\frac{\pi}{4}\right) + 2\left(\frac{\pi}{2}\right) = \pi + \pi = 2\pi$

POLYHEDRAL GAUSS-BONNET THEOREM

Let "polyhedron" P have boundary ∂P. If $\omega = \sum_{v \in P \setminus \partial P} K(v)$ and $\tau = \sum_{v \in \partial P} K(v)$, then $\omega + \tau = 2\pi\, \chi(P)$.

sum over non-boundary vertices

sum over boundary vertices

SHORTEST PATHS ON POLYHEDRA