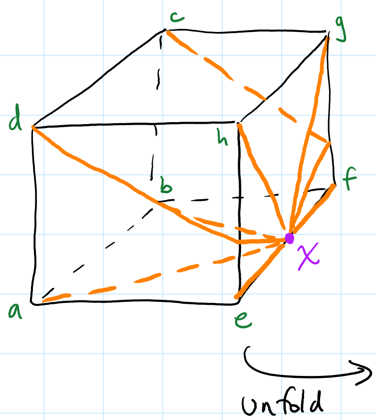
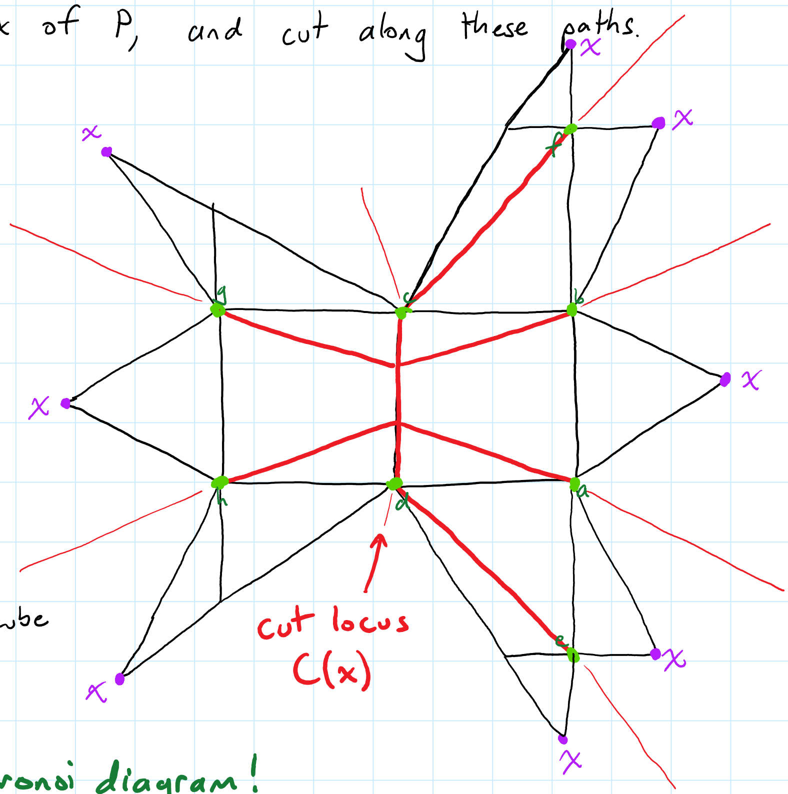
STAR UNFOLDING OF A CONVEX POLYHEDRON

Fix a point x in a convex polyhedron P. Find the shortest path from x to each vertex of P, and cut along these paths.

EXAMPLE:





NOTE: obtain a polygon with $2n$ vertices:

- n vertices of the cube
- n copies of x

The cut locus is a Voronoi diagram!

$$C(x) = \text{Vor}(\text{copies of } x \text{ in the star unfolding})$$

UNFOLDINGS SUMMARY:

	edge unfolding (net)	general unfoldings — e.g. source and star
convex polyhedra	open question	yes
nonconvex polyhedra	no	open question

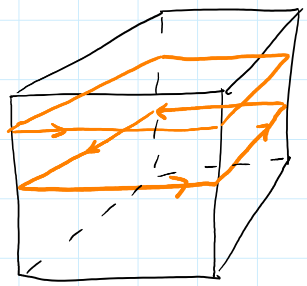
GEODESICS: path on a surface that is locally shortest

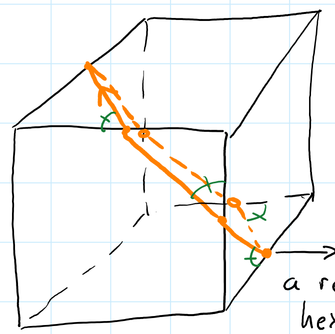
For any point x on a geodesic curve γ, there is a neighborhood of x in which γ is the shortest path between any points on γ.

- Geodesics unfold as straight lines.
- Geodesics can self-intersect. A geodesic that does not cross itself is a <u>simple</u> geodesic. A geodesic that forms a closed loop is a <u>simple closed</u> geodesic.

EXAMPLE:

simple closed geodesic





forms a regular hexagon

Most polyhedrons don't have simple closed geodesics!

Why? Recall Gauss-Bonnet Theorem:

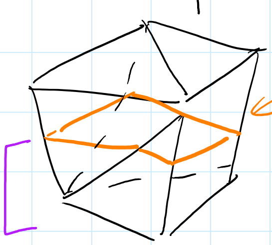
$$\omega + \tau = 2\pi \underbrace{\chi(P)}$$

total curvature at vertices of a polyhedron with boundary

total curvature along the boundary

Euler characteristic $\chi(P) = 1$ for a polyhedron with boundary an no holes

P



P'

Suppose this is a simple closed geodesic γ

γ has no curvature

P' is the lower part of P, bounded by γ

Gauss-Bonnet Theorem says: $\omega + 0 = 2\pi$

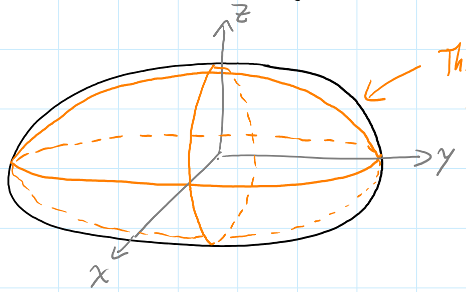
total curvature on vertices of P'

If P has a simple closed geodesic γ, then γ partitions the vertices of P into two sets, each of which has curvature that adds up to 2π.

For "most" polyhedra, this not possible.

IN CONTRAST: Any smooth surface homeomorphic to a sphere has at least 3 simple closed geodesics.

EXAMPLE: ellipsoid



The cross-sections in the coordinate planes are the simple closed geodesics.