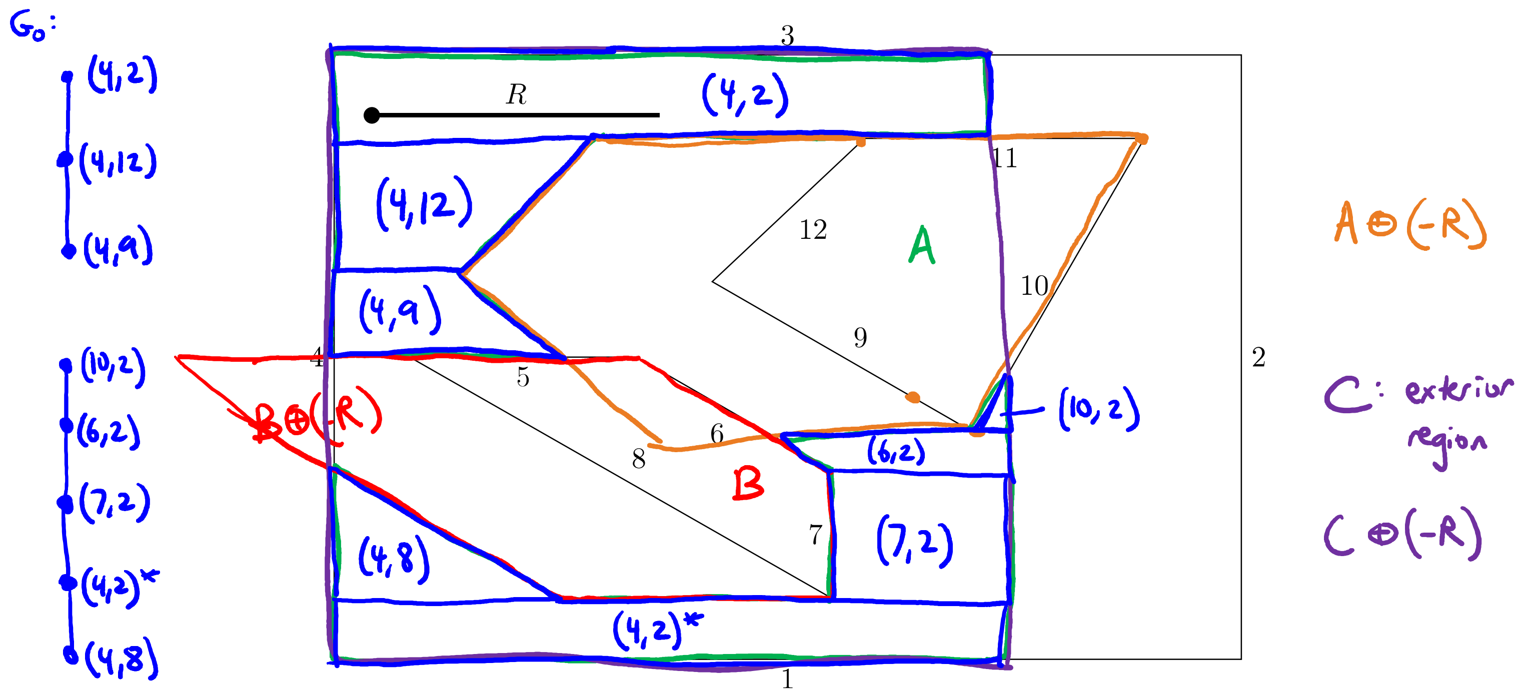
Motion Planning

Math 282 Computational Geometry

The following diagram shows a rectangular domain with two interior obstacles. The horizontal segment R with marked base point is a ladder, which is to be moved from the upper left corner to the lower right corner of the domain.



1. Suppose that ladder R may move, but it must remain horizontal and cannot intersect the boundary or obstacles. Let S_0 be the set of all possible locations of the ladder's base point. Draw S_0, and express S_0 in terms of a Minkowski Sum.

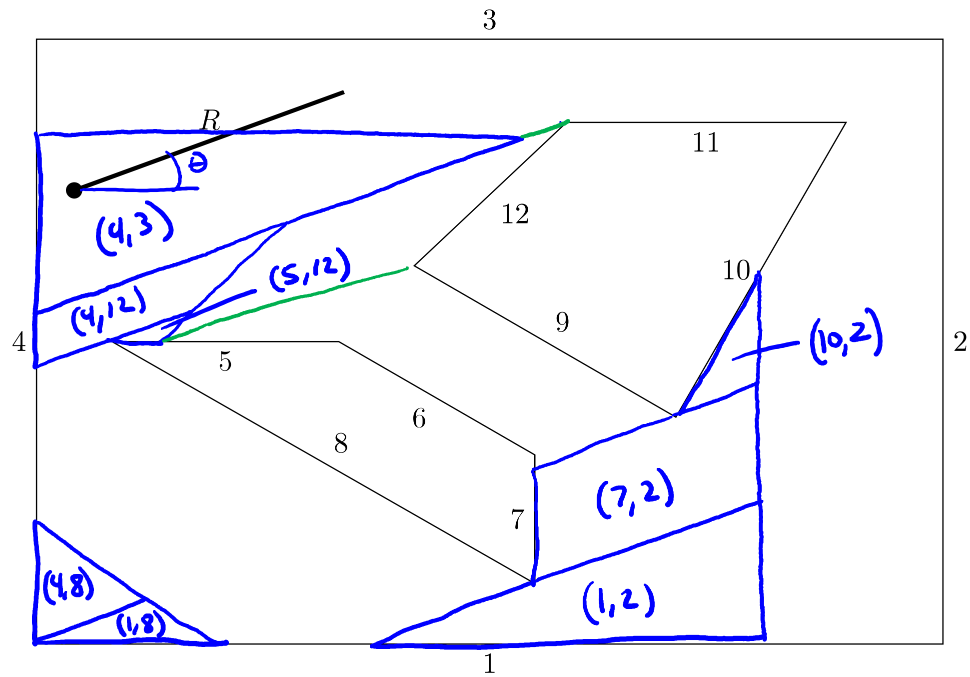
$$S_0 = \left(A \oplus (-R) \cup B \oplus (-R) \cup C \oplus (-R)\right)'$$

(complement of union of Minkowski sums)

2. Suppose the ladder's base point is placed at a point x, not on the same horizontal line as any vertex in the polygonal environment. Observe that moving the R leftward will eventually cause it to intersect an edge e_1, and moving it rightward will eventually cause it to intersect an edge e_2. Label point x with the pair $(1, 2)$. Similarly, we can label all points in S_0. Partition the set S_0 into 2-dimensional cells such that all points within each cell have the same label.

region

3. Define the *connectivity graph* G_0 as follows: each cell that you found in #2 is a node of G_0, and two nodes are connected by an edge if the boundaries of their corresponding cells intersect. Draw graph G_0.



4. Now rotate R by some angle θ. Repeat the previous construction to find a set S_θ of allowable base points. Slide the ladder parallel to its length to partition the set as before and obtain the connectivity graph G_θ.

5. How do the connectivity graphs for different values of θ relate to each other?

6. How could you join the various connectivity graphs to obtain one big graph that tell you how to move the ladder from the upper left to the lower right of the domain?