Homework 6

Math 330

Type (in LaTeX) your solutions to the following problems. Submit them either on Moodle or in the homework mailbox (RMS level 3, near the fireplace) by 4:00pm on **Thursday, October 26**.

The first four exercises are from the textbook:

1. Problem 4.2.2
Hint: the units for T_0 are Newtons.

2. Problem 4.4.3

3. Problem 4.4.6
For this problem, make sure you start with the solution to the wave equation given in equation (4.4.11). *Hint*: use $\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right]$ and a similar identity for $\sin\alpha\sin\beta$.

4. Problem 4.4.7

5. Consider the wave equation on an infinite string with $c = 1$:

$$\begin{cases} u_{tt} = u_{xx} & -\infty < x < \infty, \quad t > 0 \\ u(x,0) = f(x) & -\infty < x < \infty \\ u_t(x,0) = g(x) & -\infty < x < \infty \end{cases} \tag{$*$}$$

(a) Use D'Alembert's solution (refer to the worksheet/notes from class) to solve the wave equation $(*)$ if

$$f(x) = \begin{cases} x+1, & -1 \le x \le 0 \\ 1-x, & 0 < x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

and $g(x) = 0$. Sketch the solution for each of the following values of t: 0, $\frac{1}{2}$, 1, and $\frac{3}{2}$. Interpret your solution in terms of traveling waves.

(b) Now use D'Alembert's solution to solve the wave equation $(*)$ if $f(x) = 0$ and $g(x) = \sin(x)$. Sketch the solution for each of the following values of t: 0, $\frac{\pi}{4}$, $\frac{\pi}{2}$, $\frac{3\pi}{4}$. Interpret your solution in terms of traveling waves.