Homework 7

Math 330

Type (in LaTeX) your solutions to the following problems. Submit them either on Moodle or in the homework mailbox (RMS level 3, near the fireplace) by 4:00pm on **Thursday, October 31**.

1. Problem 4.4.3

2. Problem 4.4.6 — You should start with the solution to the wave equation given in equation (4.4.11) and show that it can be written in the form $u(x,t) = R(x-ct) + S(x+ct)$. *Hint*: use $\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$ and a similar identity for $\sin\alpha\sin\beta$.

3. Problem 4.4.7 — As with the previous problem, the text intends that you should do this problem by starting with equation (4.4.11) and applying the identities given in the hints.

For the remaining problems, consider the wave equation on an infinite string:

$$\begin{cases} \frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} & -\infty < x < \infty, \quad t > 0 \\ u(x,0) = f(x) & -\infty < x < \infty \\ u_t(x,0) = g(x) & -\infty < x < \infty \end{cases} \tag{$*$}$$

4. Use D'Alembert's solution (refer to the worksheet/notes from class) to solve the wave equation $(*)$ with $c = 1$,

$$f(x) = \begin{cases} x+1, & -1 \le x \le 0 \\ 1-x, & 0 < x \le 1 \\ 0 & \text{otherwise,} \end{cases}$$

and $g(x) = 0$. Sketch the solution for each of the following values of t: 0, $\frac{1}{2}$, 1, and $\frac{3}{2}$. Interpret your solution in terms of traveling waves.

5. Use D'Alembert's solution to solve the wave equation $(*)$ with $c = 1$, $f(x) = 0$, and $g(x) = \sin(x)$. Sketch the solution for each of the following values of t: 0, $\frac{\pi}{4}$, $\frac{\pi}{2}$, $\frac{5\pi}{4}$. Interpret your solution in terms of traveling waves.

6. Use D'Alembert's solution to solve the wave equation $(*)$ with $c = 1$, $f(x) = 0$, and $g(x) = e^{-x^2}$. Sketch the solution for each of the following values of t: $0, 1, 3, 5$. Interpret your solution in terms of traveling waves.

7. What happens if you change the value of c in the wave equation? Choose one of problems 4, 5, or 6, and experiment with different values of c. How does your choice of c affect the graphs of the solution? What is the role of c in the wave equation?