More Fourier Series

Math 330

1. Use Euler's formula to express e^{ikx} in terms of sine and cosine functions. Then do the same for e^{-ikx}.

2. Now express $\cos(kx)$ in terms of complex exponentials. Then do the same for $\sin(kx)$.

3. Use your formulas to convert a trigonometric Fourier series into a series of complex exponentials. Specifically, fill in the boxes in the following equation.

$$\frac{a_0}{2} + \sum_{k=1}^{\infty} \left(a_k \cos(kx) + b_k \sin(kx)\right) = \boxed{} + \sum_{k=1}^{\infty} \left(\boxed{}\, e^{-ikx} + \boxed{}\, e^{ikx} \right)$$

4. If $f(x) \sim \sum_{k=-\infty}^{\infty} c_k e^{ikx}$ on $[-\pi, \pi]$, then the complex Fourier coefficients are computed via

$$c_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-ikx}\, dx.$$

Compute the complex Fourier coefficients for $f(x) = e^x$. Then plot $f(x)$ together with partial sums of its complex Fourier series.

5. Let $\hat{f}(x)$ be the 2π-periodic even extension of e^x. Sketch $\hat{f}(x)$.

6. Compute the Fourier series of $\hat{f}(x)$. Plot some partial sums of this Fourier series.

7. What happens if you differentiate the Fourier series for $\hat{f}(x)$ term by term to obtain $\hat{f}'(x)$? Plot some partial sums of $\hat{f}'(x)$.

8. What happens if you differentiate the Fourier series for $\hat{f}'(x)$ term by term to obtain $\hat{f}''(x)$? Plot some partial sums of $\hat{f}''(x)$.