Continuity

MATH 348

1. What does it mean for a function to be **continuous**?

2. Which of the following functions are continuous on the specified domain?

   (a) $f(x) = x \quad$ on $(-1, 1)$

   (b) $g(x) = \frac{1}{x} \quad$ on $(-1, 0) \cup (0, 1)$

   (c) $h(x) = \sin\left(\frac{1}{x}\right) \quad$ on $(-1, 0) \cup (0, 1)$

   (d) $k(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} \quad$ on $(-1, 1)$

3. If the sequence $x_1, x_2, x_3, \ldots$ converges to some value a and the sequence $x_1, x_2, x_3, \ldots$ converges to $f(a)$, does this mean that function f is continuous at a? Why or why not?

4. If function f has the property that for *every* sequence $x_1, x_2, x_3, \ldots$ that converges to a, the sequence $f(x_1), f(x_2), f(x_3), \ldots$ converges to $f(a)$, then is f continuous at a? Why or why not?

Open Sets

MATH 348

1. If function f is continuous and A is an open set, what can you say about $f(A)$?

2. If function f is continuous and A is an open set, what can you say about $f^{-1}(A)$?

3. Is the union of every collection of open sets itself an open set?

4. Is the intersection of every collection of open sets itself an open set?

5. Is the intersection of every finite collection of open sets itself an open set?