Open Sets in $\mathbb{R}$

MATH 348

For today, all of the sets we consider will be sets of real numbers.

1. Give an example of a set of real numbers...

(a) ...that is neither open nor closed.

(b) ...that is both open and closed.

2. Give a proof or counterexample for each statement.

(a) The union of any collection of open sets is itself an open set.

(b) The intersection of any collection of open sets is itself an open set.

(c) The intersection of any finite collection of open sets is itself an open set.

3. If $f : \mathbb{R} \to \mathbb{R}$ is continuous and $A \subset \mathbb{R}$ is open, is $f(A)$ necessarily open?

4. If $f : \mathbb{R} \to \mathbb{R}$ is continuous and $A \subset \mathbb{R}$ is open, is $f^{-1}(A)$ necessarily open?

5. Can you formulate a statement about continuity in terms of *closed* sets? That is, complete the following statement.

A function $f : \mathbb{R} \to \mathbb{R}$ is continuous if and only if _____*something about closed sets*_____.

6. Let A and B be open sets and $X = A \cup B$. Let $f : A \to \mathbb{R}$ and $g : B \to \mathbb{R}$ be continuous functions that agree on $A \cap B$. Define the function $h : A \cup B \to \mathbb{R}$ by $h = f$ on A and $h = g$ on B. Is h continuous on X? Why or why not?

7. How would your answer to #6 change if A and B are *closed* sets?

8. How would your answer to #6 change if A and B are *arbitrary* sets?