Connectivity

MATH 348

1. Intuitively, which of the following spaces would you say is *connected*?

(a) $\mathbb{R}$ with the standard topology

(b) $\mathbb{R} - \{\pi\}$ with the subspace topology

(c) S^0 with the subspace topology

(d) The set $A \subset \mathbb{R}^2$ defined to be the union of the graph of $y = e^x$ and the graph of $y = 0$, with the subspace topology.

2. For each space X in #1 that is disconnected, find a separation of X.

3. Is $\mathbb{R}$ with the discrete topology connected?

4. Is $\mathbb{Z}$ with the indiscrete topology connected?

5. Prove that topological space X is connected if and only if there are no nonempty proper subsets of X that are both open and closed in X.

6. Let X be a connected topological space. Show that there is no continuous surjective map from X to S^0.

7. Let X be a disconnected topological space. Is there a continuous surjective map from X to S^0?

8. Prove that if X is connected and $f : X \to Y$ is continuous, then $f(X)$ is connected in Y.

9. What properties of sets are preserved by continuous functions? What properties are not preserved?

10. Is there a continuous surjective function from $[0, 1]$ to $(0, 1)$? Why or why not?