Quotient Spaces

MATH 348

1. Construct the torus T^2 as:

(a) A quotient of the cylinder C

(b) A quotient of $X = [0,1] \times [0,1]$

(c) A quotient of $\mathbb{R}^2$

2. What space results from identifying each pair of parallel edges of the square $[0,1] \times [0,1]$, with one edge reversed?

3. Show that if $f : X \to Y$ is a surjective continuous map that is either open or closed, then f is a quotient map.

4. Let $X = [0, 1] \cup [2, 3] \subset \mathbb{R}$ and $Y = [0, 2] \subset \mathbb{R}$. Define $f : X \to Y$ as

$$f(x) = \begin{cases} x & \text{if } x \in [0, 1] \\ x - 1 & \text{if } x \in [2, 3] \end{cases}.$$

Is f surjective? Continuous? Open? Closed? A quotient map?

5. Let $\pi_1 : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be projection onto the first coordinate. Is π_1 surjective? Continuous? Open? Closed? A quotient map?