Homotopy

MATH 348

1. Suppose T is convex and S is any topological space. Why are every two maps $f, g : S \to T$ homotopic?

2. Give an example of spaces S and T with non-homotopic functions $f, g : S \to T$.

3. Justify each of the following to show that homotopy is an equivalence relation on functions between two given topological spaces:

(a) Homotopy is reflexive.

(b) Homotopy is symmetric.

(c) Homotopy is transitive.

4. Explain which of the following pairs of spaces are homotopy equivalent.

(a) The single point space $\{0\}$ and $\mathbb{R}$

(b) The closed interval $[0, 1]$ and the single point space $\{0\}$

(c) The open interval $(0, 1)$ and the single point space $\{0\}$

(d) The annulus $A = \{(x, y) \in \mathbb{R} \mid 1 \leq \sqrt{x^2 + y^2} \leq 2\}$ and the circle S^1

(e) The punctured plane $\mathbb{R}^2 - \{(0, 0)\}$ and the circle S^1

(f) S^1 and S^0