Simplicial Complexes

MATH 348

1. Suppose the points $v_0, \ldots, v_k \in \mathbb{R}^n$ are in general position. Show that

$$\{v_1 - v_0, \quad v_2 - v_0, \quad \ldots, \quad v_k - v_0\}$$

is a linearly independent set of vectors.

2. Let $\sigma = [v_0, \ldots, v_k]$ be a simplex in $\mathbb{R}^n$. Let τ be a simplex whose vertices are some subset of $\{v_0, \ldots, v_k\}$. Show that $\tau \subseteq \sigma$.

3. Show that an n-simplex is homeomorphic to a closed n-dimensional ball.

4. Let σ be a k-simplex.

   (a) How many subsimplices does σ have?

   (b) How many faces does σ have?

5. Describe the boundary of a simplex in terms of barycentric coordinates.

6. Consider the three conditions in the definition of a simplicial complex. Give an example of a subspace K of $\mathbb{R}^n$ and a finite list of simplices that satisfies:

   (a) Properties 1 and 2, but not 3.

   (b) Properties 1 and 3, but not 2.

   (c) Properties 2 and 3, but not 1.

7. Let S and T be simplices of a simplicial complex K. Show that $S \cap T$ is either empty or a subsimplex of both S and T.