Euler Characteristic

MATH 348

1. Draw several planar graphs and compute their Euler characteristics. What does the Euler characteristic tell you about a planar graph?

2. Extend problem #1 to planar two-dimensional simplicial complexes. What does the Euler characteristic tell you about such complexes?

3. Explain why the Euler characteristic of a simplicial complex K can also be written

$$\chi(K) = \sum_{\sigma \in K} (-1)^{\dim(\sigma)}$$

where the sum is over all simplices σ in K, and $\dim(\sigma)$ is the dimension of σ.

4. Compute $\chi(S^2)$ using at least two different triangulations of the sphere S^2.

5. Compute $\chi(T^2)$ using at least two different triangulations of the torus T^2.