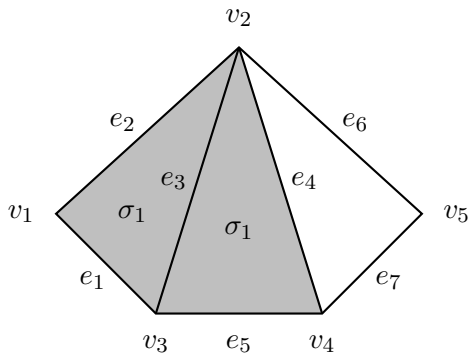
Introduction to Simplicial Homology

MATH 348

Consider the following simplicial complex K. Note that K consists of two 2-simplices, seven 1-simplices, and five 0-simplices. Moreover, K has one *hole*—the triangle that is not filled in.*



1. Write three one-dimensional loops that *do not* encircle the hole. Write each loop as a sum of 1-simplices.

2. Let γ_1 and γ_2 be loops that you wrote in #1. Consider the difference $\gamma_1 - \gamma_2$ with $\mathbb{Z}_2$ coefficients. What can you say about this difference?

3. Write three one-dimensional loops that encircle the hole. Write each loop as a sum of 1-simplices.

4. Let γ_1 and γ_2 be loops that you wrote in #3. Consider the difference $\gamma_1 - \gamma_2$ with $\mathbb{Z}_2$ coefficients. What can you say about this difference?

*source: Michael Starbird and Francis Su, *Topology Through Inquiry*, Section 15.2.