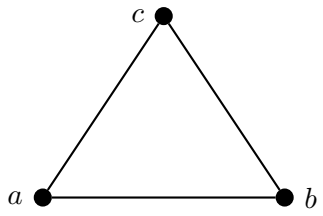
Introduction to Simplicial Homology

MATH 348

1. Consider the following simplicial complex K.



(a) What are C_0 and C_1? What are their dimensions?

(b) What is the dimension of Z_1? (Hint: What is the only nontrivial 1-cycle?)

(c) What is B_1?

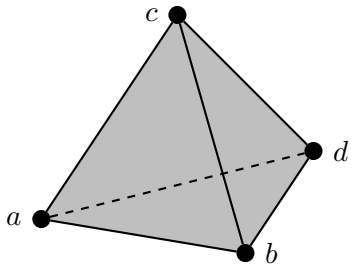
(d) What vector space is $H_1(K)$? What is its dimension?

(e) What is the dimension of $\operatorname{Im} \delta_1$? (Hint: rank-nullity theorem)

(f) What vector space is $H_0(K)$? What is its dimension?

(g) Why is $H_i(K) = 0$ for all $i > 1$?

2. Consider the following simplicial complex K, which is a *hollow tetrahedron*.



(a) What are the dimensions of C_i for all nonnegative integers i?

(b) What is the dimension of $\operatorname{Im} \delta_1$?

(c) What is the dimension of $H_0(K)$?

(d) What are the dimensions of $\operatorname{Ker} \delta_1$ and $\operatorname{Im} \delta_2$?

(e) What is the dimension of $H_1(K)$?

(f) What are the dimensions of $\operatorname{Ker} \delta_2$ and $\operatorname{Im} \delta_3$?

(g) What is the dimension of $H_2(K)$?