Exam 1 Information

MATH 348, Fall 2024

In topology, it's essential to be fluent with definitions, to recall examples, to understand key properties, to know the proofs of core theorems, and to be able to apply this knowledge. This exam will focus on the definitions, examples, properties, and theorems from Sections 2.1 through 4.2 of Crossley's *Essential Topology* text, and related material from class.

Books, notes, and internet-capable devices will not be permitted during the in-class exam. Calculators will be allowed, but probably not useful, since the problems will require very little arithmetic. Exam will be written to be completed in 60 minutes, though you may have the full 80-minute class period. The St. Olaf Honor Code applies to this exam.

Be sure that you can state precise definitions for the following concepts:

- ϵ-δ definition of a continuous function at a point
- continuous function on a set
- open and closed intervals in $\mathbb{R}$
- open and closed sets in $\mathbb{R}$ (definition from Chapter 2)
- preimage of a set S under a function f, denoted $f^{-1}(S)$
- topological space (the four axioms), a topology on a set
- open ball in $\mathbb{R}^n$
- continuous function from one topological space to another
- discrete topology, indiscrete topology, subspace topology
- basis for a topology
- surjective map (or surjection)
- disconnected space, connected space
- bounded function, bounded set (in $\mathbb{R}$)
- open cover, finite refinement, compact set

Be sure that you can recall or construct examples such as the following:

- An intersection of open sets might not be open
- Examples of topologies on finite sets
- Examples of spaces with the subspace topology
- Examples of continuous and discontinuous functions on topological spaces
- Examples of bases for various topologies
- Examples of disconnected and connected spaces

Be sure that you can write proofs of the following results. (Understanding the ideas and logical structure of a proof is much better than memorizing the proof verbatim.)

- Proof of Proposition 2.6 in the text
- Proof of Proposition 3.9 in the text
- Proof of Proposition 3.44 in the text
- Proof of Lemma 4.3 in the text
- Proof that if X is connected and $f : X \to Y$ is continuous, then $f(X)$ is connected in Y