Exam 2 Information

MATH 348, Fall 2024

This exam will focus on the definitions, examples, properties, and theorems from Sections 4.1 through 6.3 of Crossley's *Essential Topology* text, and related material from class. Concepts from Chapters 2 and 3 are also relevant, as they are necessary to understand the concepts in Chapters 4–6.

Books, notes, and internet-capable devices will not be permitted during the in-class exam. Calculators will be allowed, but probably not useful, since the problems will require very little arithmetic. Exam will be written to be completed in 60 minutes, though you may have the full 80-minute class period. The St. Olaf Honor Code applies to this exam.

Be sure that you can state precise definitions for the following concepts:

- Disconnected space, connected space
- Bounded function, bounded set (in $\mathbb{R}$)
- Open cover, finite refinement, compact set
- Hausdorff property
- Homeomorphic, homeomorphism
- Open map
- Disjoint union
- Cartesian product, product topology, product space
- Quotient space, quotient topology, quotient map
- Equivalence relation
- Klein bottle
- Homotopy, homotopic maps, homotopy classes
- Homotopy equivalence, homotopy equivalent spaces
- Contractible space

Be sure that you can recall or construct examples illustrating the following concepts:

- Disconnected and connected spaces
- Compact and non-compact spaces, and how these relate to bounded functions
- Hausdorff and non-Hausdorff spaces
- Homeomorphic and non-homeomorphic spaces
- How the concept of homeomorphic spaces relates to the concepts of connectedness, compactness, and the Hausdorff properties
- Disjoint unions of topological spaces
- How disjoint unions relate to the concepts of connectedness, compactness, and the Hausdorff

properties

- Product spaces
- How product spaces relate to the concepts of connectedness, compactness, and the Hausdorff properties
- Quotient spaces
- How quotient spaces relate to the concepts of connectedness, compactness, and the Hausdorff properties
- Homotopic and non-homotopic maps
- Homotopy equivalent spaces, and spaces that are not homotopy equivalent
- How the concept of homotopy equivalence relates to the concepts of homeomorphism, connectedness, compactness, and the Hausdorff properties

Be sure that you can write proofs of the following results. (Understanding the ideas and logical structure of a proof is much better than memorizing the proof verbatim.)

- Proof of Lemma 5.4 in the text (page 57)
- Proof of Lemma 5.27 in the text (page 66)
- Proof of Theorem 5.35 in the text (page 70)
- Proof of Proposition 6.5 in the text (page 94)
- Proof of Lemma 6.11 (page 97). Also know whether the converse of this lemma is true or false, and why.