Final Exam Information

MATH 348, Fall 2024

This exam will focus on the definitions, examples, properties, and theorems that we have studied this semester. The exam will be cumulative, with emphasis on the topics we have studied in November and December.

Books, notes, and internet-capable devices will not be permitted during the in-class exam. Calculators will be allowed, but probably not useful, since the problems will require very little arithmetic. The St. Olaf Honor Code applies to this exam.

Be sure that you can state precise definitions for the following concepts:

- Open and closed sets in $\mathbb{R}$ (definition from Chapter 2)
- Preimage of a set S under a function f, denoted $f^{-1}(S)$
- Topological space (the four axioms), a topology on a set
- Continuous function from one topological space to another
- Discrete topology, indiscrete topology, subspace topology
- Basis for a topology
- Disconnected space, connected space
- Open cover, finite refinement, compact set
- Hausdorff property
- Homeomorphic, homeomorphism
- Open map
- Cartesian product, product topology, product space
- Quotient space, quotient topology, quotient map
- Equivalence relation
- Homotopy, homotopic maps, homotopy classes
- Homotopy equivalence, homotopy equivalent spaces
- Contractible space
- Degree or winding number of a map $f : S^1 \to S^1$
- Simplex, simplicial complex, subsimplex, face, boundary, interior
- Euler number or Euler characteristic
- Triangulation (of a topological space)
- Surface, orientable, non-orientable
- n-chain, boundary operator, cycles, boundaries
- Homology vector space, homologous, Betti numbers
- Čech complex, Vietoris-Rips complex

Be sure that you can recall or construct examples illustrating the following concepts:

- Examples of topologies on finite sets
- Bases for various topologies
- Disconnected and connected spaces
- Compact and non-compact spaces, and how these relate to bounded functions
- Hausdorff and non-Hausdorff spaces
- How the concept of homeomorphic spaces relates to the concepts of connectedness, compactness, and the Hausdorff properties
- How disjoint unions relate to the concepts of connectedness, compactness, and the Hausdorff properties
- How product spaces relate to the concepts of connectedness, compactness, and the Hausdorff properties
- How quotient spaces relate to the concepts of connectedness, compactness, and the Hausdorff properties
- Homotopic and non-homotopic maps
- How the concept of homotopy equivalence relates to the concepts of homeomorphism, connectedness, compactness, and the Hausdorff properties
- How the Euler number (or Euler characteristic) is independent of any particular triangulation of a topological space
- How the the concept of Euler number relates to the concepts of homotopy equivalence and homoemorphism
- How Betti numbers of a simplicial complex can be computed from boundary matrices
- How the Čech and Vietoris-Rips complexes can be constructed from a point cloud in $\mathbb{R}^n$

Be sure that you can write proofs of the following results. (Understanding the ideas and logical structure of a proof is much better than memorizing the proof verbatim.)

- Proof of Proposition 2.6 in the text (page 10)
- Proof of Proposition 3.44 in the text (page 32)
- Proof of Lemma 4.3 in the text (page 38; proof on page 37)
- Proof of Lemma 5.4 in the text (page 57)
- Proof of Proposition 6.5 in the text (page 94)
- Proof of Lemma 6.11 (page 97). Also know whether the converse of this lemma is true or false, and why.

In addition, the exam will contain one of the following reflection questions.[1]

- What mathematical ideas are you curious to know more about as a result of taking this class? Give one example of a question about the material that you'd like to explore further, and describe why this is an interesting question to you.
- How has your mathematical imagination been enhanced as a result of taking this class? Give at least three examples.
- Give one example of a mathematical idea from this class that you found creative, and explain what you find creative about it. For example, you can choose an instance of creativity you experienced in your own problem-solving, or something you witnessed in another person's definition or reasoning.

[1] Questions by Francis Su, `https://www.francissu.com/post/7-exam-questions-for-a-pandemic-or-any-other-time`