Homework 2

MATH 348

due at 5pm on Tuesday, September 24, 2024

Solve the following problems and communicate your solutions clearly using complete sentences. Your proofs may rely on definitions and theorems stated in the text or given in class.

Remember what the syllabus says about appropriate collaboration, and document what sources you use and what assistance you receive as you work on this homework.

For this second homework, you must type your solutions to at least the first five problems in LaTeX. You may type or write your solutions to the remaining problems. Make sure your solutions are easy to read, in order, and clearly labeled. Upload a single file containing your solutions to the Homework 2 assignment on Moodle.

Some of the problems will be graded in detail, and the rest will be graded for completion.

1. (5 points) Let $X = \{a, b, c, d, e\}$ be a set with 5 elements, with topology given by

$$\mathcal{T} = \{\varnothing, \{a\}, \{b\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\}, \{a, b, c, d, e\}\}.$$

What are the open sets in the subspace topology for the subset $Y = \{a, c, e\}$?

2. (6 points) Exercise 3.5 in the text — Specifically, show that the collection of open sets in Example 3.11 satisfies the four rules in the definition of a topological space.

3. (6 points) Exercise 3.6 in the text

4. (6 points) Exercise 3.8 in the text

5. (7 points) Exercise 3.11 in the text

6. (6 points) Let $\mathcal{T}$ be the collection of all subsets U of $\mathbb{R}$ such that $\mathbb{R} - U$ is either finite or all of $\mathbb{R}$.

(a) Show that $\mathcal{T}$ is a topology on $\mathbb{R}$.

(b) Give an example of a function $f : \mathbb{R} \to \mathbb{R}$, other than the identity function, that is continuous with respect to this topology. Explain how you know that your function is continuous.

7. (4 points) Determine which of the following collections of subsets of $\mathbb{R}$ are bases for some topology:

(a) $\mathcal{C}_1 = \{(n, n+2) \subset \mathbb{R} \mid n \in \mathbb{Z}\}$

(b) $\mathcal{C}_2 = \{[a, b] \subset \mathbb{R} \mid a < b\}$

(c) $\mathcal{C}_3 = \{[a, b] \subset \mathbb{R} \mid a \leq b\}$

(d) $\mathcal{C}_4 = \{(a, b] \subset \mathbb{R} \mid a < b\}$