Homework 3

MATH 348

due at 5pm on Tuesday, October 1, 2024

Solve the following problems and communicate your solutions clearly using complete sentences. Your proofs may rely on definitions and theorems stated in the text or given in class.

Remember what the syllabus says about appropriate collaboration, and document what sources you use and what assistance you receive as you work on this homework.

For this homework, you must type your solutions to all of the problems in LaTeX. You may include hand-drawn diagrams in your solutions. Make sure your solutions are easy to read, in order, and clearly labeled. Upload a single file containing your solutions to the Homework 3 assignment on Moodle.

Some of the problems will be graded in detail, and the rest will be graded for completion.

1. (4 points)

(a) Is every topology a basis? Explain why or why not.

(b) Is every basis a topology? Explain why or why not.

2. (4 points) Give examples of subsets A and B in $\mathbb{R}^2$ such that:

(a) A and B are connected, but $A \cap B$ is not.

(b) A and B are connected, but $A - B$ is not.

(c) A is connected, B is disconnected, and $A \cap B$ is connected.

(d) A and B are disconnected, but $A \cup B$ is connected.

You do not need to prove anything for this problem. Examples without proof will suffice.

3. (5 points) Exercise 4.1 in the text

4. (6 points) Exercise 4.3 in the text

5. (4 points) Give an example demonstrating that an arbitrary union of compact sets in a topological space X is not necessarily compact.

6. (5 points) Exercise 4.5 in the text

7. (6 points) Exercise 4.6 in the text — Only show that $\mathrm{GL}(3, \mathbb{R})$ is not compact.

8. (6 points) Let $\mathcal{B} = \{(-a, a) \mid a \in \mathbb{R}\}$.

(a) Show that $\mathcal{B}$ is the basis for a topology on $\mathbb{R}$.

(b) Describe the open sets in the topology generated by basis $\mathcal{B}$.

(c) Is $\mathbb{R}$ with the topology generated by $\mathcal{B}$ a connected space? Why or why not?